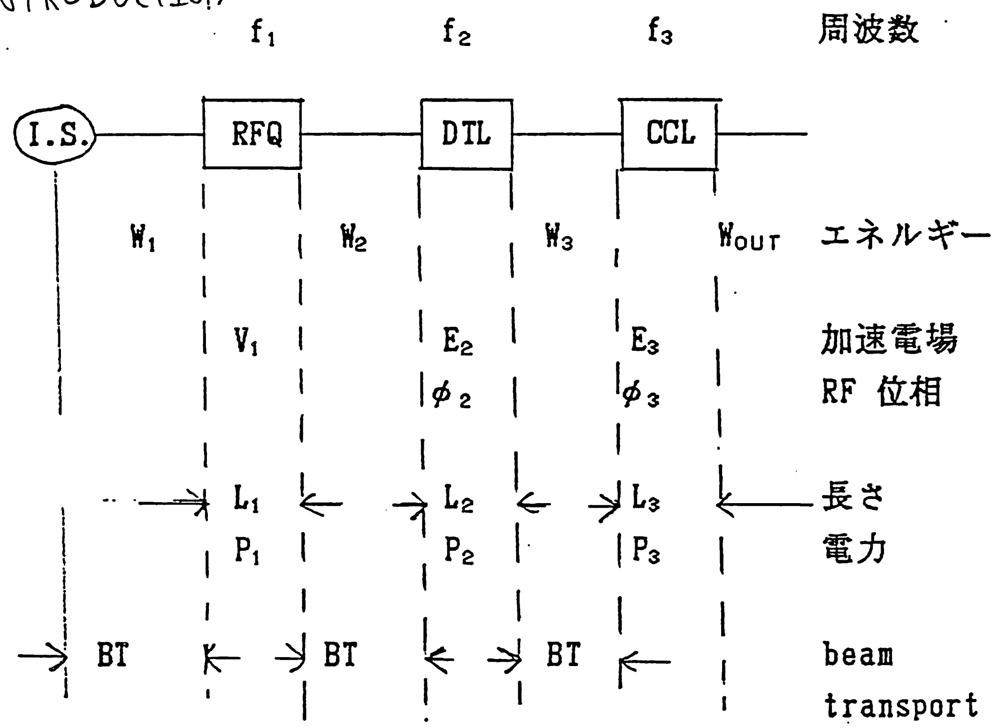
1 GeV リニアックのビームダイナミックス

870710 T.Kato and Y.Yamazaki

* 50 keV ---> 1 GeV
* H^- (p)
* 20 mA, 400 μsec, 50 pps

$\left\{ \begin{array}{l} f_1 \doteqdot f_2 \\ f_2 = 2f_1 \end{array} \right\} \quad f_3 = 3 \times f_2$

1. INTRODUCTION



なにを考えてこれらのパラメーターを選ぶか

stable, low cost, maintenance free, beam loss etc.

どこに難しさを感じるか

リニアックの性能をチェックする為の諸量

入出力エネルギーが与えられている時

Pc	高周波電力	cost optimization
L	長さ	

longitudinal acceptance　　longitudinal motion
transverse acceptance　　transverse motion

emittance growth (tr. and lon.)

beam dynamics under some errors of E_0, ϕ and Q-magnet

PROTON	ELECTRON
β = v/c 変化	β = 1 一定

50 keV	0.01	1 MeV	0.046	3 MeV	0.08
150 MeV	0.51	1000 MeV	0.88		

cell length $\beta\lambda$, $\beta\lambda/2$

400 MHz	3 MeV	$\beta\lambda$ =	5.98 cm
	150 MeV	$\beta\lambda$ =	37.97 cm
1200 MHz	150 MeV	$\beta\lambda/2$ =	6.33 cm
	1000 MeV	$\beta\lambda/2$ =	10.93 cm

transit time factor $T = T(\beta) = \dfrac{\int E(z)\cos kz\,dz}{\int E(z)\,dz}$

$k = 2\pi/\beta\lambda$

shunt impedance $Z = E_0^2 / (P_c/L)$, $E_0 = \dfrac{1}{L}\int E(z)\,dz$

P_c = dissipation power in a cell, $L = \beta\lambda$

electronになじみの人々は

$E_0 T \cos\varphi \to E$

$ZT^2 \to Z_{eff}$

と使っていることがある.

加速効率

$$Z = E_0^2 / (P_c/L)$$

effective shunt impedance

$$Z_{eff} = ZTT = \frac{(E_0 T L \cos\varphi)^2}{P_c L \cos^2\varphi} = \frac{(\Delta V)^2}{P_c L \cos^2\varphi} \quad \text{for a cell}$$

リニアック全体に対して

$$ZTT = \frac{V^2}{P_c L \cos^2\varphi}$$

or $$P_c = V^2/(ZT^2 L \cos^2\varphi)$$

$V = E_0\ T\ L \cos\phi$ ここで E_0, T, ϕ は平均値、L = total length

$$P_c = \frac{V E_0}{Z T \cos\varphi} \qquad \therefore P_c \propto E_0$$

$Z \propto f^{1/2}$ だから $\boxed{P_c \propto f^{-1/2}}$

example 150 MeV -----> 1000 MeV, E_0 = 3 MV/m

600 MHz , a = 2.54 cm ----> Pc = 86.9 MW

1200 MHz , a = 1.5 cm ----> Pc = 63.6 MW

(86.9 / $\sqrt{2}$ = 61.4 MW)

$$V = \sqrt{Z\ T\ T\ L\ P_c\ \cancel{\cos 2\phi}\ \cos^2\varphi} \propto \sqrt{ZT^2}$$

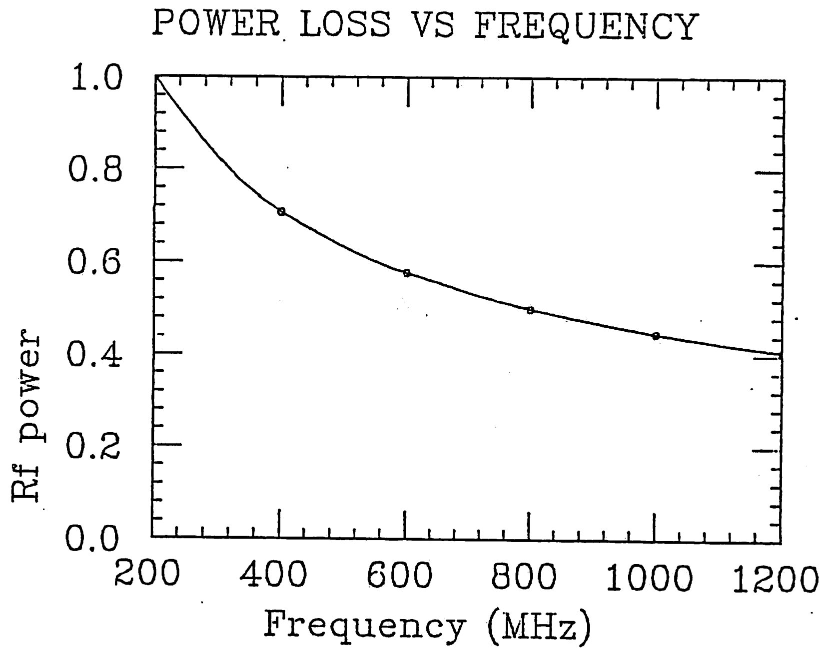

POWER LOSS VS FREQUENCY
Rf power
1.0
0.8
0.6
0.4
0.2
0.0
200
400
600
800
1000
1200
Frequency (MHz)

Table 1 リニアックの主なパラメーターと他との比較　　86年12月

	CCL-1	CCL-2	CCL-3	single	EHF	
FREQUENCY	600	1200	600	500	1200	MHz
DTL FREQUENCY	200	400	200	200	400	MHz
RFQ FREQUENCY	200	200 OR 400	200	100	50 AND 400	MHz
Win	150	150	100	100	150	MeV
Wout	1000	1000	1000	1000	1200	MeV
E0	3.3	4.0	3.0	3.0	5.6	MV/m
Pc	95.4	90.3	89.7	100*	143	MW
TANK LENGTH	328	272	371	458**	232	m
STABLE PHASE	30	30	26	26	57-25	degree
AVERAGE ZTT	31	39	30	22	41	MΩ/m
TANK NUMBER	136	152	154	208		
CELL NUMBER	1784	2954	2082	1664		

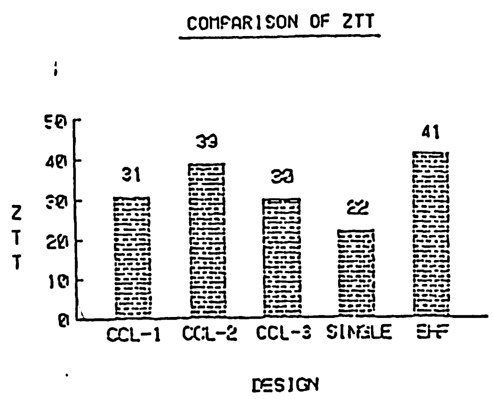


Fig.15 各デザインの ZTT の比較.

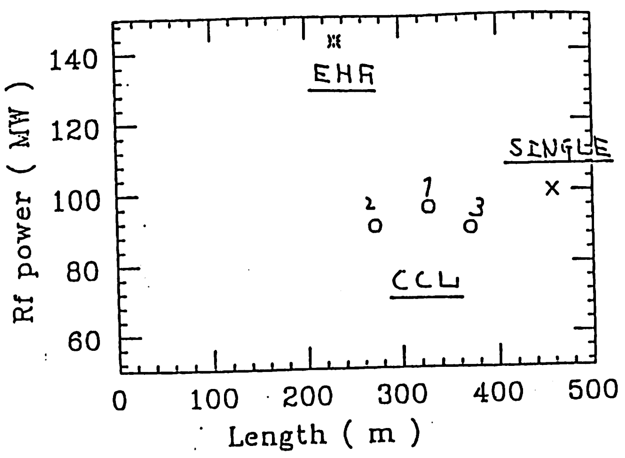


Fig.16 各デザインの長さと励振電力の比較.

Design tools

cavity calculation ----> SUPERFISH from LOS ALAMOS

RFQ ------> QKEK design, beam simulation and vane construction

DTL ------> modified PARMILA from LOS ALAMOS

DTL focusing design ----> DELIFO

1. constant phase advance without rf defocusing
2. constant phase advance with rf defocusing
3. constant beam radius
4. constant tune depression

CCL ------> PROEND

1. constant β structure
2. variable β structure
   including three kinds of focusing method
   singlet, doublet and triplet

Beam transport ----> TRIRFQ and TRIPLE

beam transport from RFQ to DTL, and DTL to CCL

Debuncher or prebuncher ----> DEBUN

Linear chain model for a post stabilized DTL ---> TOMOR

Linear chain model for CCL on PC9801

参考文献

1. 1 GeV リニアックの試み KEK ASN - 259 (1986) October.
2. 1 GeV リニアックの試み (2) KEK ASN - 265 (1987) February.
3. 1 GeV リニアックの試み (3) KEK ASN - 271 (1987) April.
4. RFQ acceleration up to a few MeV proton energy, GEMINI design report, KEK,p.465 (1987).

2. LONGITUDINAL MOTION

$$\frac{d^2}{dz^2}(\Delta\phi) = \frac{2\pi\, g\, E_0 T \sin\phi_s}{\lambda \beta^3 \gamma^3 mc^2}(\Delta\phi)$$

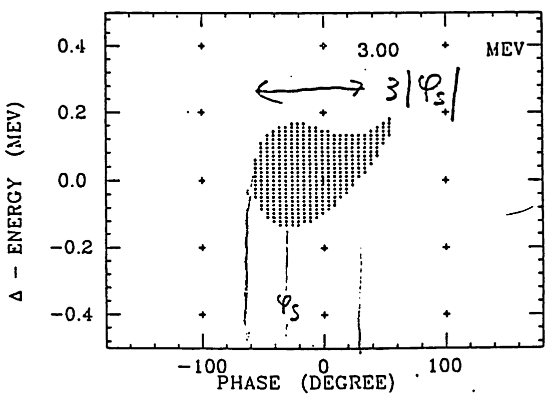
$\Delta\phi = \phi - \phi_s$, $\Delta W = W - W_s$ 単振動

$$\Delta W_{max} = \pm \gamma \sqrt{\frac{2}{3\pi}\frac{mc^2}{\gamma} E_0 T (\beta\gamma)^3 \phi_s^3 \lambda}$$

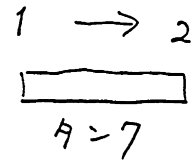
3 MV/m	ϕs = -30	-26	
W = 750 keV	51	41	keV
2 MeV	112	90	
3 MeV	156	126	
4 MV/m 150 MeV	2.2 MeV		

$\Delta\phi_{max} = 3/2 * \phi s$

DTL acceptance



damping



$$\Delta\phi_1 \Delta W_1 = \Delta\phi_2 \Delta W_2$$

$$\frac{\Delta\phi_2}{\Delta\phi_1} = \frac{\Delta W_1}{\Delta W_2}$$

$$= \left[\frac{(\beta_1\gamma_1)^3 E_1 T_1 \sin\phi_1}{(\beta_2\gamma_2)^3 E_2 T_2 \sin\phi_2}\right]^{1/4}$$

	β	γ	T	
3 MeV	0.08	1.0	0.74	
150	0.51	1.16	0.63	0.82
1000	0.88	2.07		0.81

DTL 3 MeV ----> 150 MeV

$$\frac{\Delta\phi_2}{\Delta\phi_1} \sim 0.23 \qquad \frac{\Delta W_2}{\Delta W_1} \sim 4.3$$

750 keV → 150 MeV

$$\frac{\Delta\phi_2}{\Delta\phi_1} \sim 0.13$$

CCL 150 MeV ---> 1000 MeV

$$\left(\frac{1.32\ \mathrm{MeV}}{0.24} \sim 5.5\right)$$

by コンピュータ

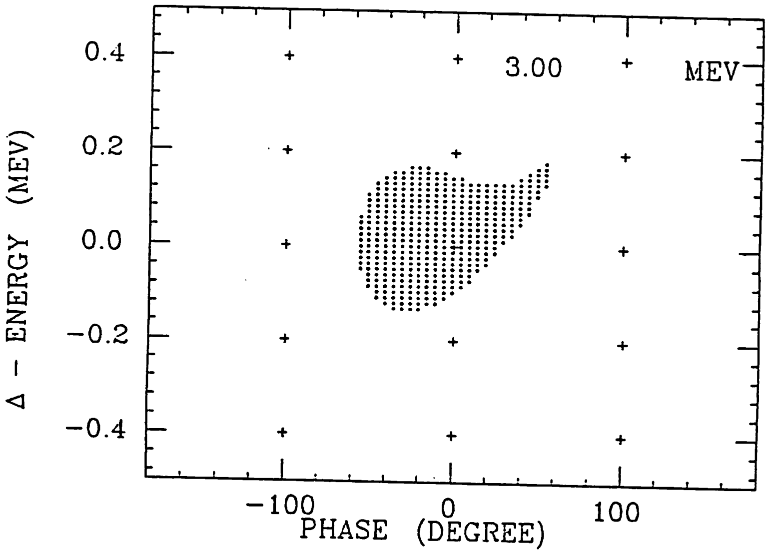
$$\frac{\Delta\phi_2}{\Delta\phi_1} \sim 0.43 \qquad \frac{\Delta W_2}{\Delta W_1} \sim 2.3$$

PHASE OSCILLATION

$$\frac{2\pi}{\lambda_1} = \sqrt{-\frac{2\pi q E T \sin\phi}{\lambda \beta^3 \gamma^3 m c^2}}$$

	150 MeV	1000 MeV
4 MV/m	13.6 m	73.4 m
λ_1, f	11.2 MHz	3.6 MHz

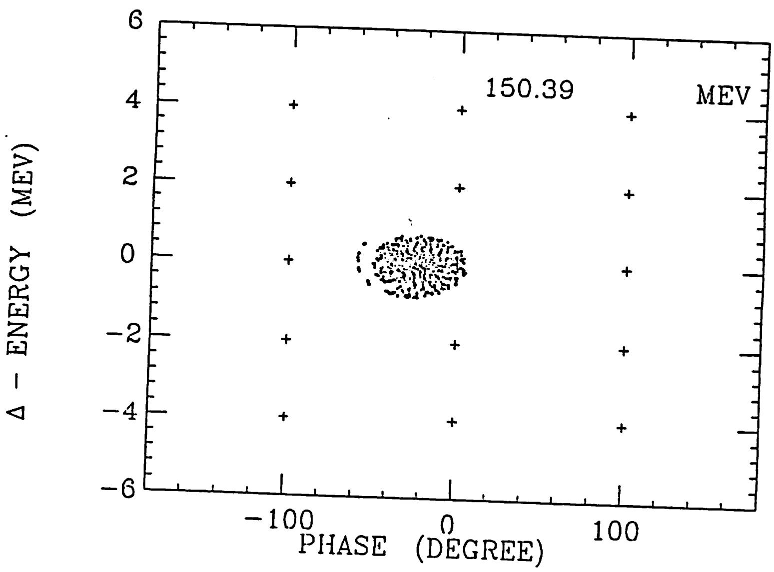
DTL acceptance



90% full
240 keV

計算式
251 keV

3 MV/m
$\varphi_s \sim 26°$



90% full
1.32 MeV

CCL

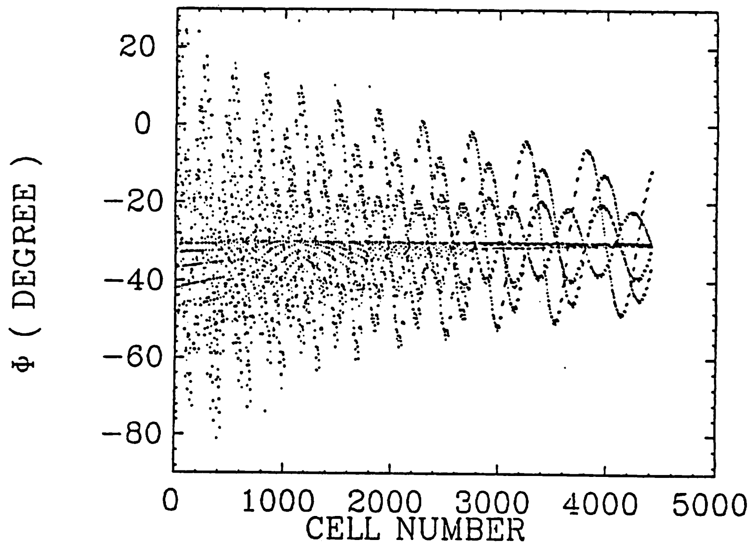
3 MV/m

3660~4280セル

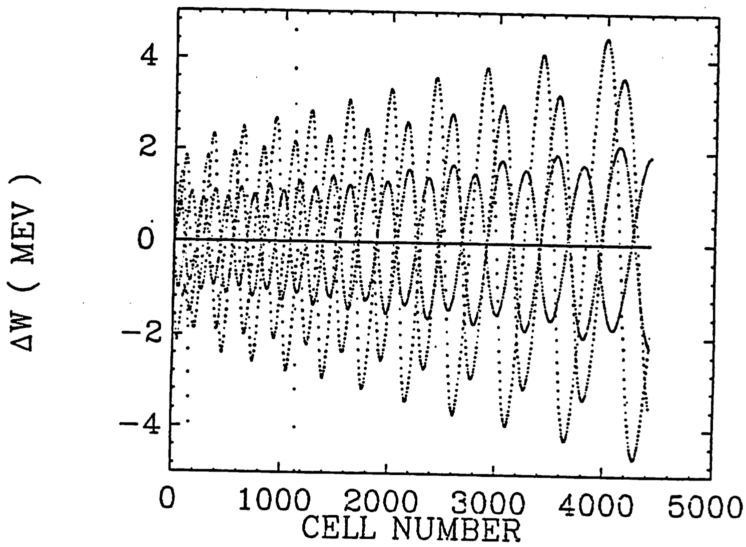
λ = 67 m

(計算で ~79 m)

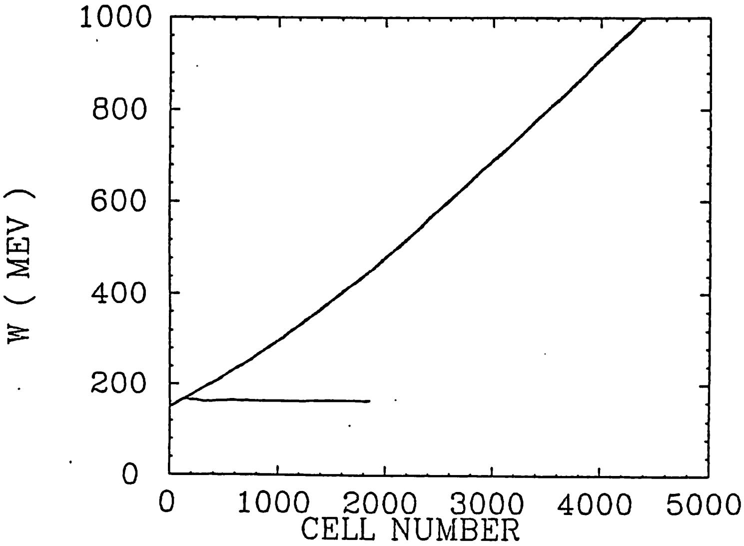
Φ — NCELL



ΔW — NCELL



W — NCELL



CCL acceptance

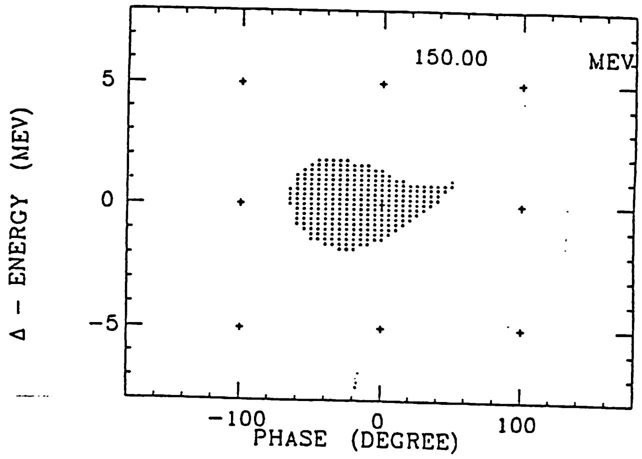


Fig.8 Longitudinal acceptance in the constant β structure. E = 3 MV/m. 12 short tanks.

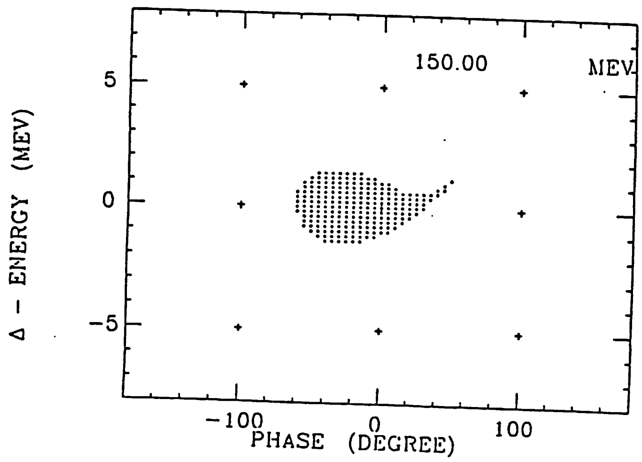
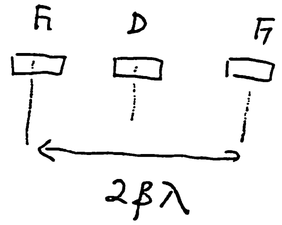


Fig.9 Longitudinal acceptance in the variable β structure. E = 3 MV/m. 12 short tanks.

3, transverse motion

$$\frac{d^2 x}{ds^2} + \left(k_t^2(s) L^2 + \Delta\right) x = 0$$

$$s = \frac{z}{L}, \qquad L = 2\beta\lambda$$



$$k_t^2 = \frac{q}{mc^2}\,\frac{cB'}{\beta\gamma}$$

$$\Delta = \Delta_{rf} + \Delta_{sc}$$

$$\Delta_{rf} = \frac{q}{mc^2}\,\frac{\pi E_0 T \sin\varphi}{\lambda \beta^3 \gamma^3} L^2 \quad (\propto \sin\varphi)$$

$$\Delta_{sc} = -\frac{3 Z_0}{8\pi}\,\frac{q}{mc^2}\,\frac{I\lambda}{\beta^2\gamma^3}\,\frac{1-f(p)}{r^2 b} L^2$$

$Z_0 = 377\,\Omega$, 仮定 $b = r$, $f(p) = \frac{1}{3}$

$$b = \frac{|\phi_s|}{2\pi}\beta\lambda$$

$$\theta_0^2 = k_t^2 L^2 X$$

$$X = \frac{4}{\pi}\sin\frac{\pi\Lambda}{2}$$ focusing efficiency factor

$$\Lambda = \frac{L_q}{L}$$ L_q = Q-magnet length

$$\mu_0^2 = \frac{\theta_0^4}{8\pi^2} + \Delta_{rf}$$ phase advance for zero current

$$\mu^2 = \frac{\theta_0^4}{8\pi^2} + \Delta_{rf} + \Delta_{sc}$$ phase advance for current I

$$\mu_t = \frac{-\Delta_{sc}}{\mu_0^2}$$ transverse space charge parameter

$$\left\{ \begin{aligned} \mu^2 &= \mu_0^2 (1 - \mu_t) \quad , \\ \mu_t &= 1 - \frac{\mu^2}{\mu_0^2} \end{aligned} \right.$$

$$\frac{\mu}{\mu_0} = \text{tune depression}$$

$$\psi = \frac{1 + \left(\frac{\theta_0}{2\pi}\right)^2}{1 - \left(\frac{\theta_0}{2\pi}\right)^2} \left(= \frac{r_+}{r_-} = \sqrt{\frac{\beta_+}{\beta_-}} \right)$$ modulation factor

◎ $$\frac{\Delta_{sc}}{\Delta_{rf}} \sim \frac{240 \pi^2 I}{\beta^2 \lambda |\varphi|^3 E_0 T \sin|\varphi|}$$

	β	Δ_{sc}/Δ_{rf}	Δ_{sc}
750 keV	0.04	0.3	0.07
2 MeV	0.065	0.1	0.016
3	0.08	0.06	0.009
150	0.507	0.004	

◎ $$\frac{\Delta\mu/\omega^2}{k_t^2} = \frac{q}{mc^2}\,\frac{X}{\sqrt{2}}\,\frac{\lambda}{\pi\beta\gamma^3}\,E_0 T \sin|\varphi|$$

但し $\mu_0 = 90^\circ$ ($\Delta\mu = 0$ のとき) とした.

400 MHz, $\lambda = 0.75$, $E_0 = 3$ MV/m , $\Lambda = \frac{1}{2}$

	β	T	$\Delta\mu/\omega^2/k_t^2$
750 keV	0.04	0.62	0.0038
2 MeV	0.065	0.70	0.0026
3 MeV	0.08	0.74	0.0022
150 MeV	0.51	0.82	0.0001

transverse acceptance

$$A = \frac{a^2 \mu}{\psi \ell} \left(= \frac{a^2}{\beta_t} \right)$$

$$\ell = 2\beta\lambda \quad (\text{for DTL})$$

400 MHz DTL, $a = 0.75$ cm, 3 MeV injection
$\Lambda = 1/2$

μ	θ_0^2	ψ	A_n (π cm mr)	B' (kG/cm)
90°	14	2.09	2.81	27
60°	9.3	1.62	2.43	18
70.3°	10.9	1.76	2.61	21

PARMILA beam simulation

70.3°			2.36	

$$\begin{cases} k_t^2 = \dfrac{e}{mc^2} \dfrac{cB'}{\beta\gamma} \\ \theta_0^2 = k_t^2 L^2 \dfrac{4}{\pi} \sin\dfrac{\pi\Lambda}{2} \end{cases}$$

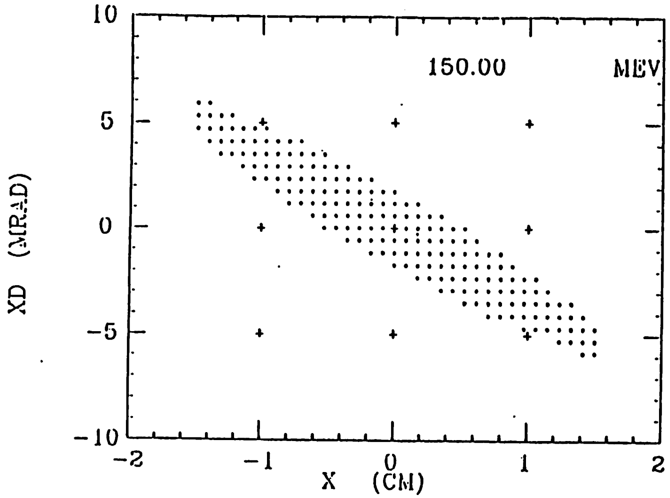


Fig.16 x - x' acceptance.
H = 2.33 ^{k}G/cm.
0 short tanks.

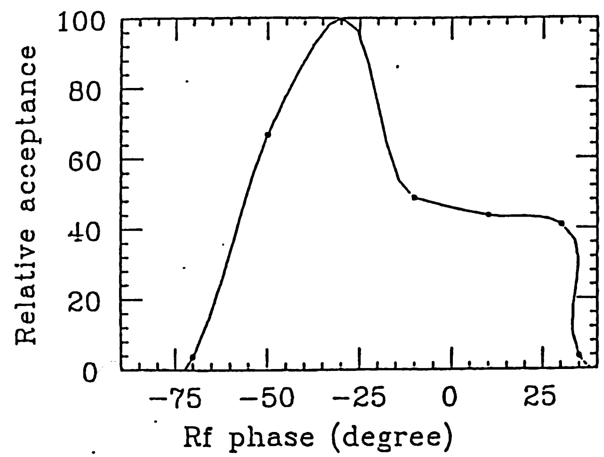


Fig.6 Relarive variation of CCL transverse acceptance vs. rf phase.

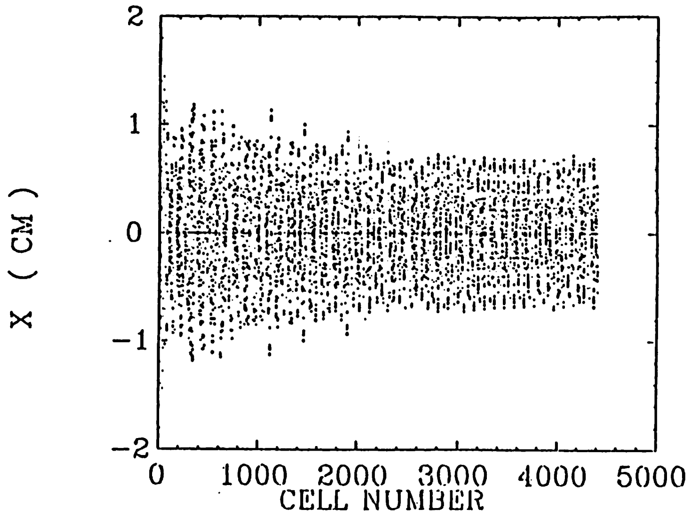


Fig.17 x motion in the SCL.
H = 1.81 ^{k}G/cm.

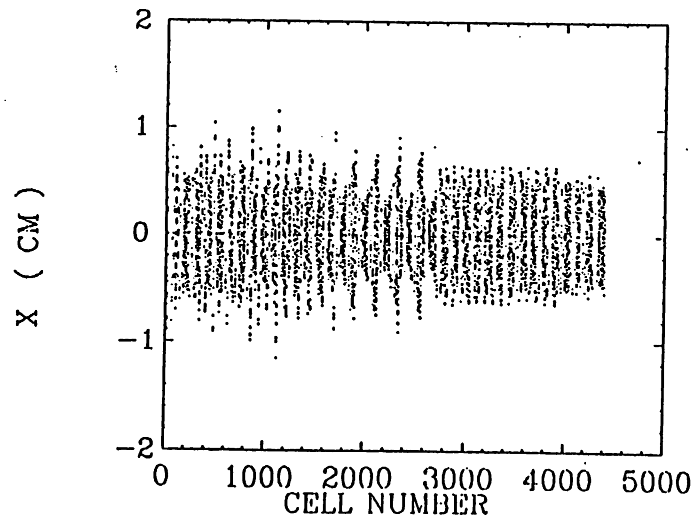


Fig.18 x motion in the SCL.
H = [illegible]85 ^{k}G/cm.

3-5

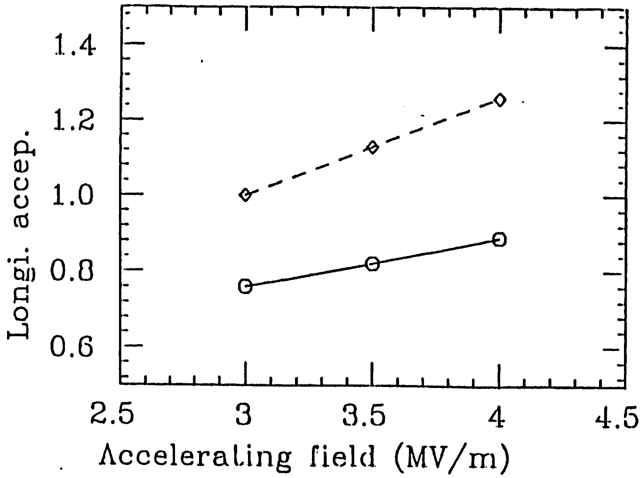


Fig.10 Relative longitudinal acceptance vs. accelerating field.
dashed line --- const. β
solid line --- variable β

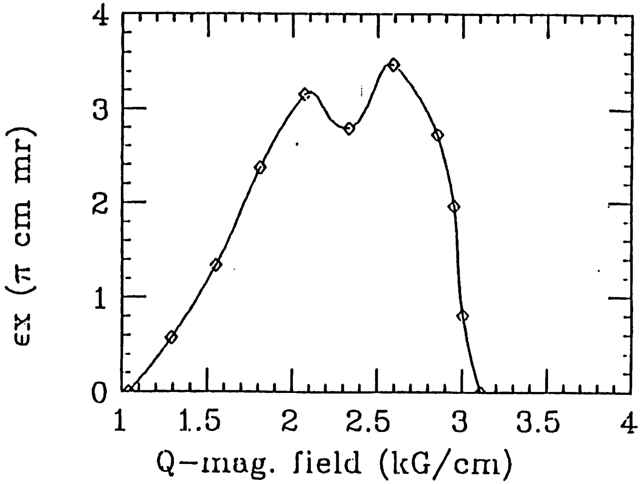


Fig.13 Transverse acceptance vs. first Q-magnetic field.

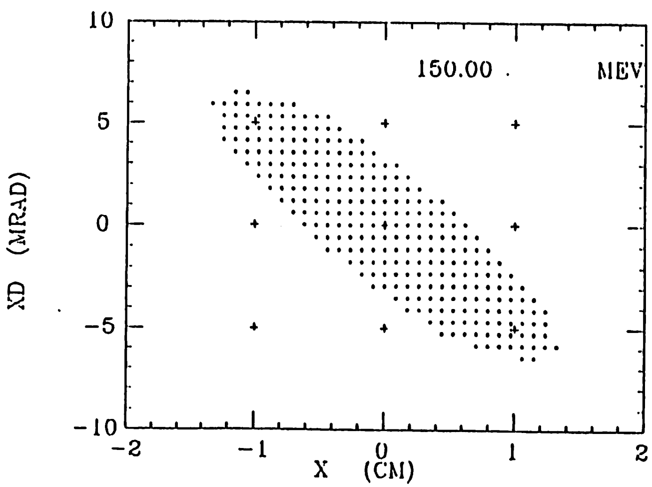


Fig.14 x - x' acceptance.
$H = 1.81^{K}G/cm$.
12 short tanks.

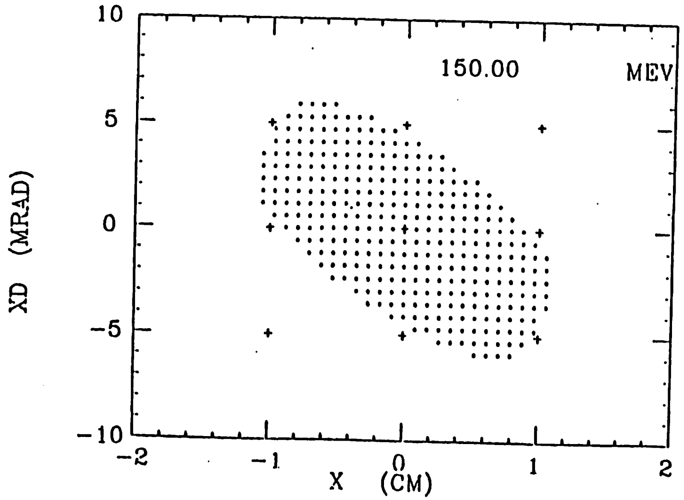


Fig.15 x - x' acceptance.
$H = 2.33^{K}G/cm$.
12 short tanks.

RFQ acceptance

$$\theta_0^2 = B = \frac{8\lambda^2}{mc^2}\frac{XV}{a^2}$$

$X = 1 - A I_0(ka)$ = focusing efficiency parameter

$$\mu_0^2 = \frac{\theta_0^4}{8\pi^2}, \quad \psi = \frac{1+\theta_0^2/4\pi^2}{1-\theta_0^2/4\pi^2}$$

$$A = \frac{a^2\mu}{\psi L}$$

$L = \beta\lambda$

◎ Aを書きなおすと

$$A_x = \frac{1}{2\sqrt{2}\pi}\frac{a^2}{\beta\lambda}\frac{B(4\pi^2-B)}{4\pi^2+B}$$

$A_{x\,max}$ は $B = 4(\sqrt{2}-1)\pi^2 = 16.35$ の時である。

β として (3 MeV) 0.08 をとると

a	0.1	0.2	0.3	cm
A_x	0.10	0.41	0.91	π cm mr
V	27.3	109	245	kV
V/a	27.3	55	82	MV/m

$E_{KL} = 19.4$ MV/m

$\mu_0 = 90° \longrightarrow B = 2\sqrt{2}\pi\mu_0 = 13.96$ であり、

B は もっと小さくても かまわない。

space charge を考える current limit (の最大値) は B~6 付近にある。

$a = 0.3$ cm , $\beta\lambda = 6$ cm (3MeV)

B	4	6	8	10	
ψ	1.23	1.36	1.51	1.68	
μ_0	0.45 (26°)	0.675 (39°)	0.90 (52°)	1.13 (65°)	
$A_{x,n}$	0.44	0.60	0.72	0.81	π cm·mr
V	60	90	120	150	kV
V/a	20	30	40	50	MV/m

(E_{KU} = 19.4 MV/m)

(注)

X = 1 とおくのは あますぎるかも。

∵ β大の所では { $A' \sim 0.4 \sim 0.5$; $X \sim 0.6$ 程度

故に

V を $\frac{1}{0.6} \sim 1.7$ 倍に しなければ いけない

$\frac{V}{a} = \frac{E_S}{\sqrt{X}}$

◎ $\theta_0^2 = B =$ constant の場合

$$\frac{\lambda^2 \times V}{a^2} = \text{const.}$$ となる

($R_0 = \frac{a}{\sqrt{X}}$ ~~= average~~ radius)

$$A \propto \frac{a^2}{\beta\lambda}$$

◎ Kilpatrick field limit に対する表面電場の大きさを一定に保つ場合
周波数によらず B は一定とすると

$$A \propto \frac{\lambda^2}{\beta}$$

(但し $E_{KL} \propto \sqrt{f}$ とする)

	400 MHz	200 MHz
a	2	6 mm
B	6	6
V	40	92 kV
V/a	20	15 MV/m
E_{KL}	19.4	14.7 MV/m
$A_{x,n}$	0.27	1.2 π cm mr
	1.98 KL	1.96 EKL

$\mu = 0.675$, $\psi = 1.358$, $\beta = 0.08$

$\frac{V}{a} = \frac{E_s}{\sqrt{X}} = 15$ MV/m

$\therefore E_s = 21.2$ MV/m

$E_{max} = 1.36 E_s = 28.8$ MV/m $= 1.96\, E_{KL}$

$\frac{V}{a} = \frac{E_s}{\sqrt{X}} = 20$ MV/m

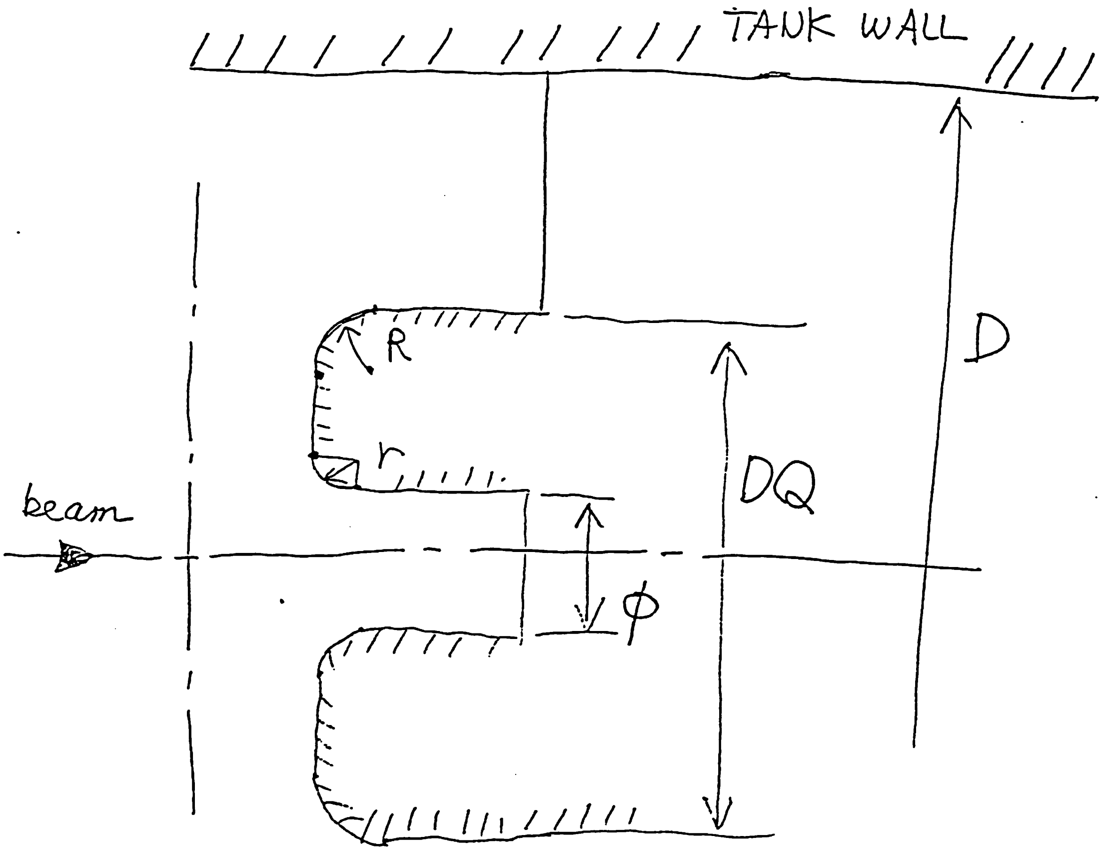
$E_s = 28.3$ MV

$E_{max} = 38.47$ MV

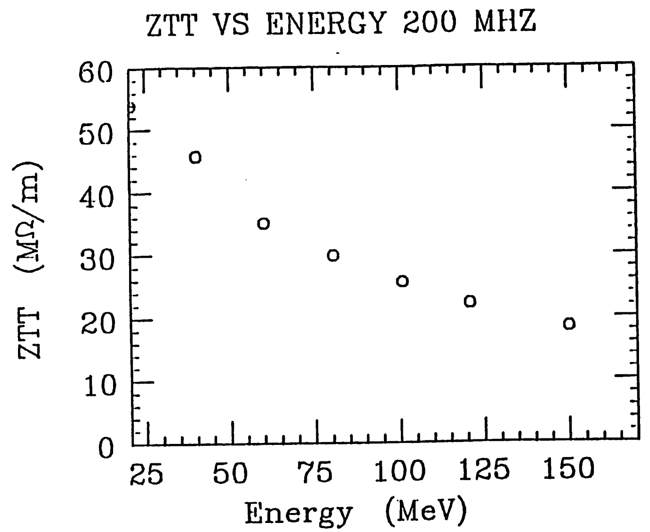
~~Et~~ $= 1.98$

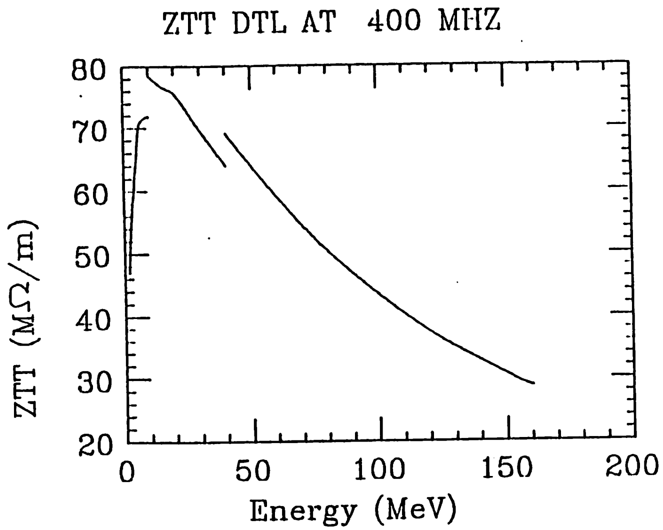
~~$\sqrt{X} = 0.35$~~ $X = 0.5792$

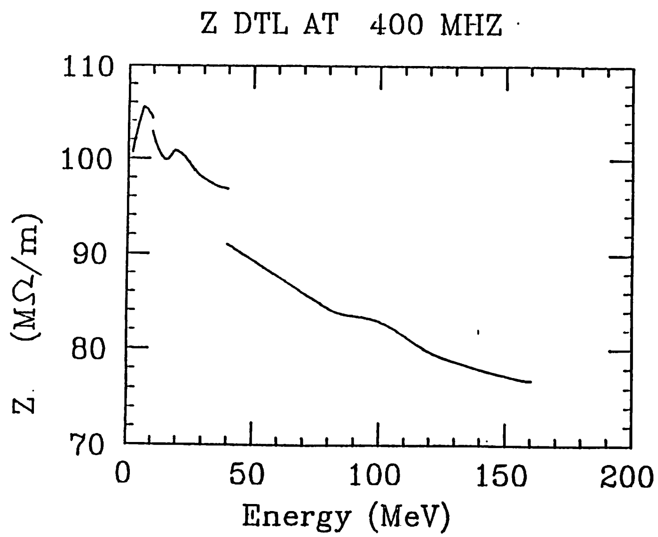
4. DTL GEOMETRY AND CCL GEOMETRY

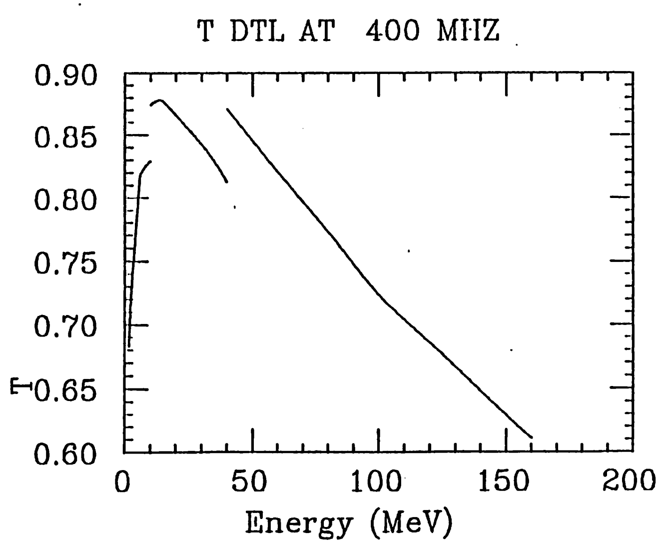


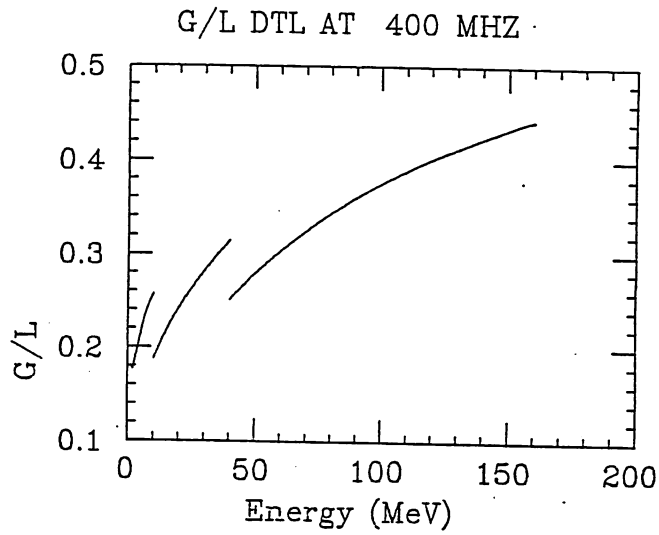
ENERGY (MeV)	D	DQ	ϕ	R	r	
3 - 10	47	9	1.5	(2)	0.5	cm
10 - 40	45	8	1.5	2	0.5	
40 - 150	43	7	1.5	2	0.5	

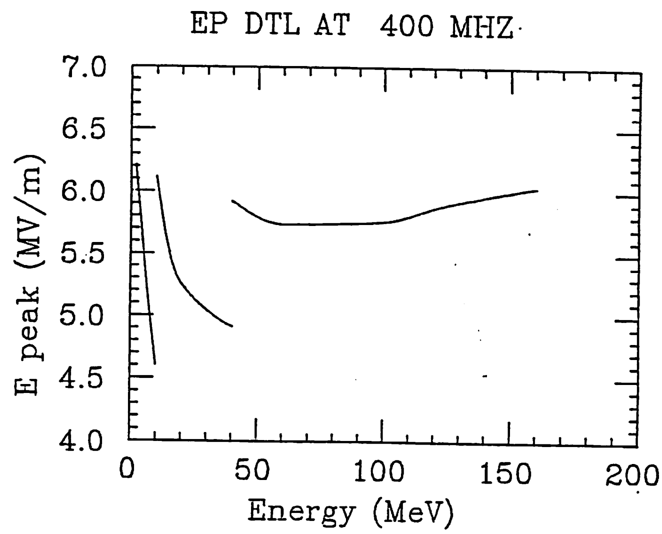

ZTT VS ENERGY 200 MHZ
ZTT (MΩ/m)
60
50
40
30
20
10
0
25
50
75
100
125
150
Energy (MeV)

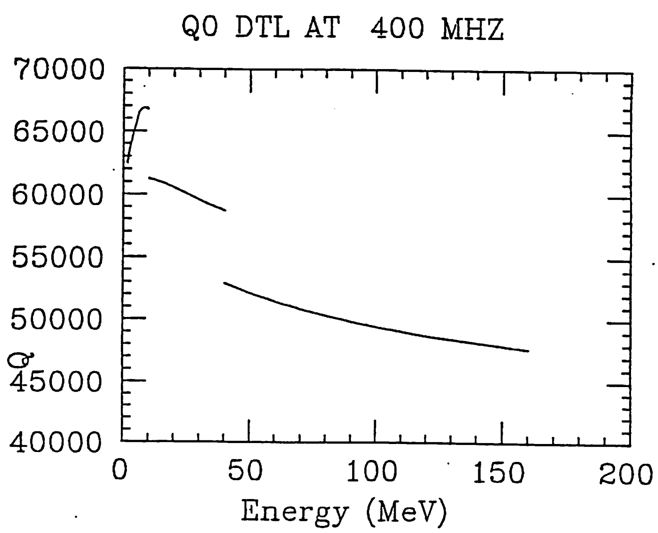

ZTT DTL AT 400 MHZ
ZTT (MΩ/m)
80
70
60
50
40
30
20
0
50
100
150
200
Energy (MeV)


Z DTL AT 400 MHZ
Z (MΩ/m)
110
100
90
80
70
0
50
100
150
200
Energy (MeV)

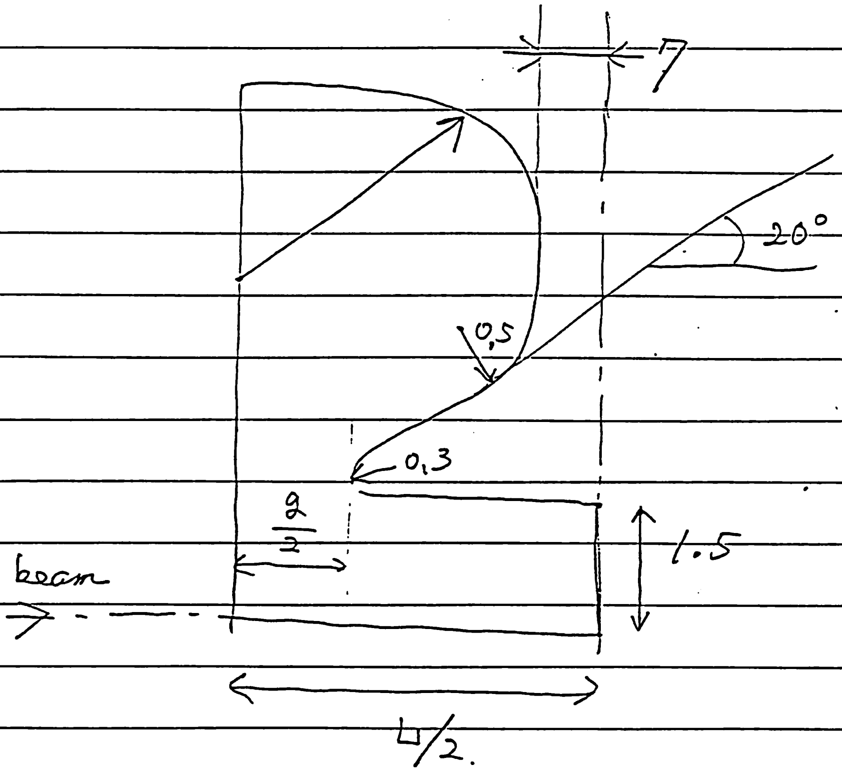

T DTL AT 400 MHZ
T
0.90
0.85
0.80
0.75
0.70
0.65
0.60
0
50
100
150
200
Energy (MeV)


G/L DTL AT 400 MHZ
0.5
0.4
0.3
0.2
0.1
G/L
0
50
100
150
200
Energy (MeV)


EP DTL AT 400 MHZ
7.0
6.5
6.0
5.5
5.0
4.5
4.0
E peak (MV/m)
0
50
100
150
200
Energy (MeV)


Q0 DTL AT 400 MHZ
70000
65000
60000
55000
50000
45000
40000
Q
0
50
100
150
200
Energy (MeV)

CCL geometry

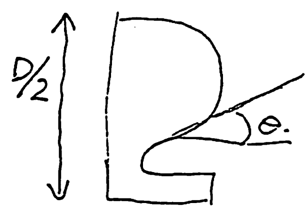


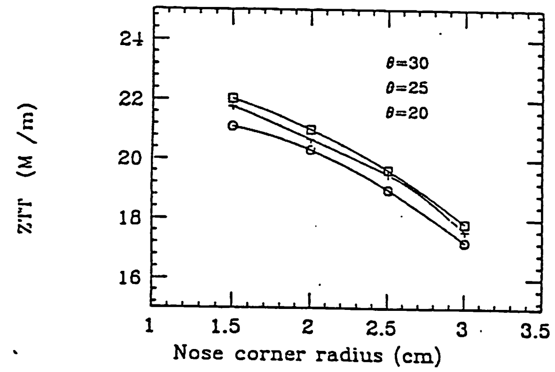
f.= 1200 MHz, g/L を変えて ZT^2 を最大にする

87. 3. 31

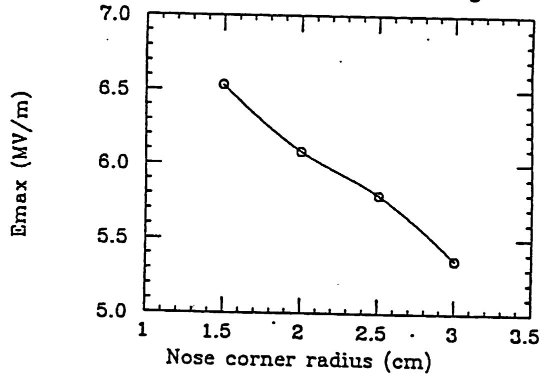
空洞の形 nose cone の角度

θ	20°	30°	
Z	56.60	49.78	
ZT^2	45.50	40.04	
D	19.27	16.38	cm
Ep	6.1	5.9	MV/m

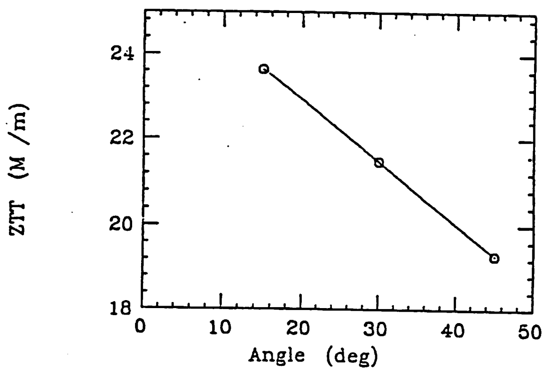




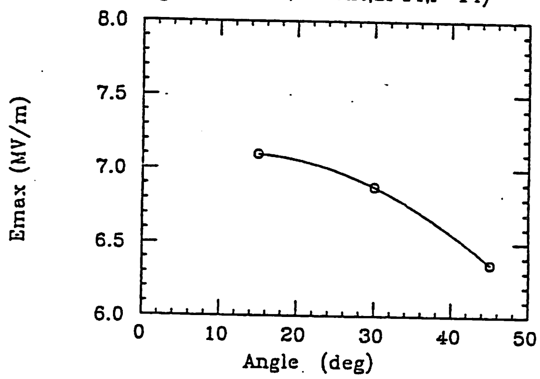
Nose radius vs Emax,re-ent, 30 deg



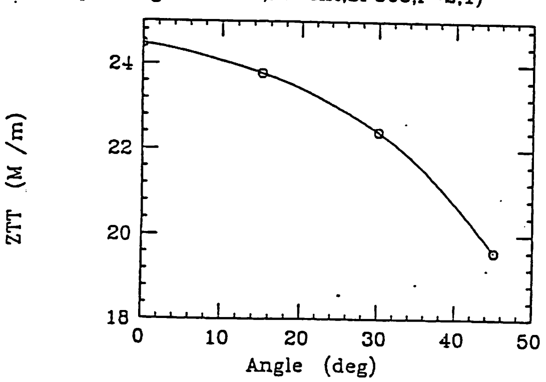
Taper angle vs ZTT,re-ent,SP14,r=14)

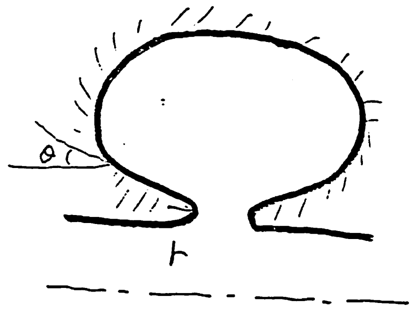


Taper angle vs Emax,re-ent,SP14,r=14)

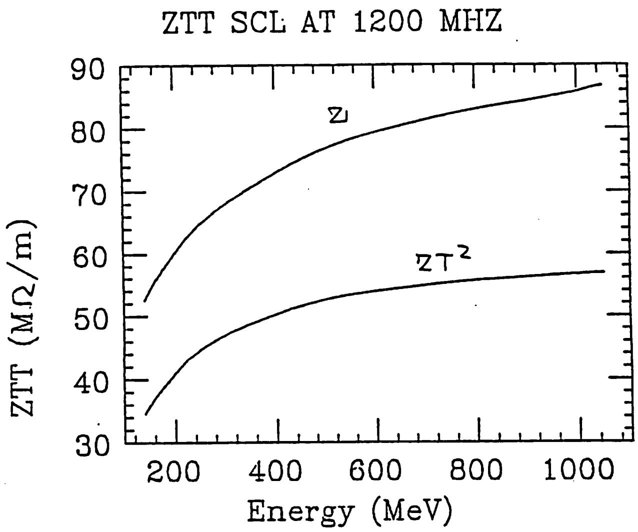


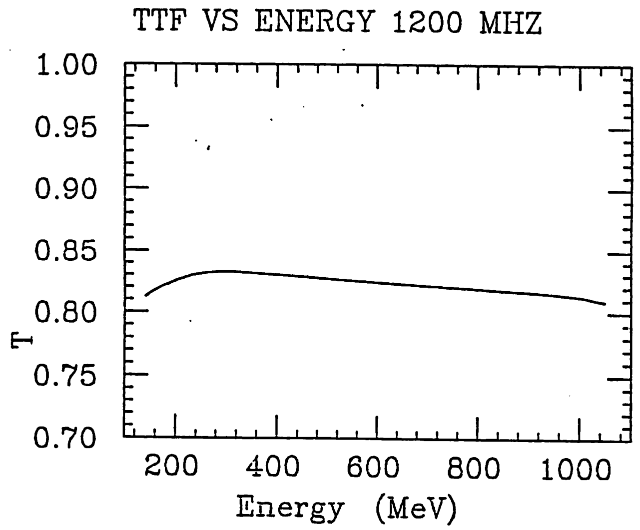
Taper angle vs ZTT,re-ent,SP308,r=2,1)

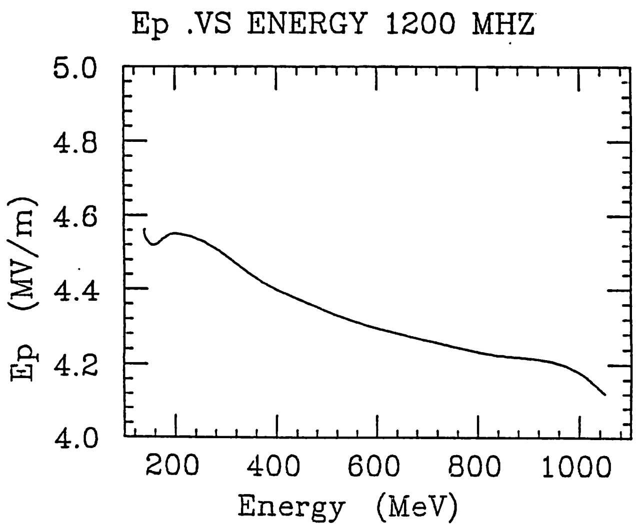


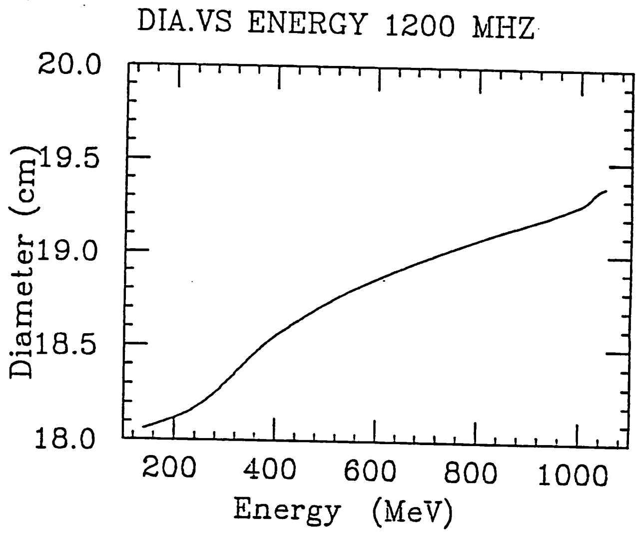


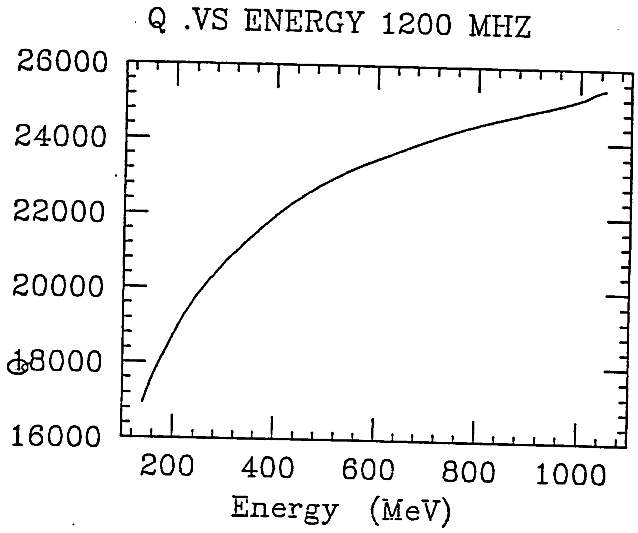
(200MHz KEK DEBUNCHER)

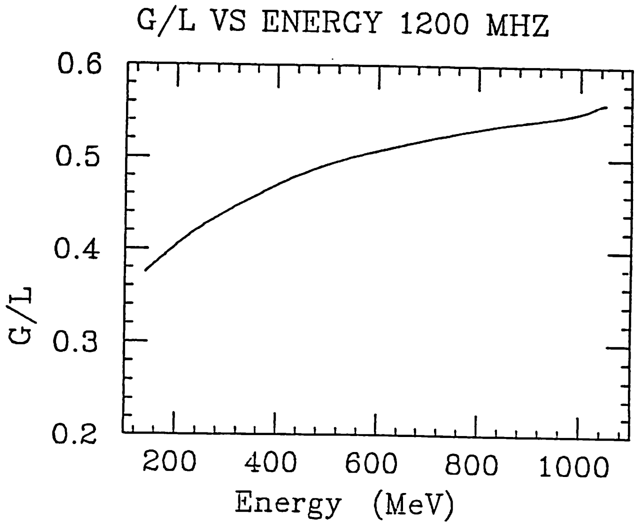

ZTT SCL AT 1200 MHZ
Z
ZT²
ZTT (MΩ/m)
90
80
70
60
50
40
30
200
400
600
800
1000
Energy (MeV)


TTF VS ENERGY 1200 MHZ
T
1.00
0.95
0.90
0.85
0.80
0.75
0.70
200
400
600
800
1000
Energy (MeV)


Ep .VS ENERGY 1200 MHZ
Ep (MV/m)
5.0
4.8
4.6
4.4
4.2
4.0
200
400
600
800
1000
Energy (MeV)


DIA.VS ENERGY 1200 MHZ
Diameter (cm)
20.0
19.5
19.0
18.5
18.0
200
400
600
800
1000
Energy (MeV)


Q .VS ENERGY 1200 MHZ
Q
26000
24000
22000
20000
18000
16000
200
400
600
800
1000
Energy (MeV)


G/L VS ENERGY 1200 MHZ
G/L
0.6
0.5
0.4
0.3
0.2
200
400
600
800
1000
Energy (MeV)

5. CCL タンクの長さの選び方

1. 製作可能な長さ

2. available rf power

3. transverse focusing
   focusing period

4. longitudinal motion
   longitudinal acceptance
   phase slip

5. mode spacing

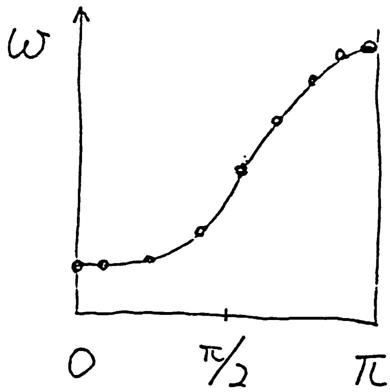
MODE SPACING OF $\pi/2$ MODE

$$\Delta\omega/\omega \sim \gamma\pi/2N$$

γ = coupling coefficient
N = total number of cells

$Q_L = \omega/\Delta\omega$ だから

$$N < \gamma\pi Q_L / 2$$



SUPERFISH CALCULATION

$Q_0 \sim 17000 - 25000$

1/1.3

$Q_0 \sim 13000 - 19000$

$\gamma = 0.05$, $Q_L = Q_0/2$ とすれば

$N \sim 550 - 750$

余裕をみこんで N max = 400 とすれば
main cell = 200 となる

at 150 MeV ----- 13 m

at 1000 MeV ---- 22 m

KILPATRICK FIELD LIMIT

$$f = 1.643\,E^2 \exp(-8.5/E)$$

200 MHz	14.7 MV/m	600 MHz	23.0 MV/m
400	19.4	800	26.0
430	20.0	1200	31.0
500	21.3	1290	32.0

ちまた にいわれる許容限界

RFQ	2.5 - 3	倍
DTL, CCL	2	倍

これは条件によるのでは ?

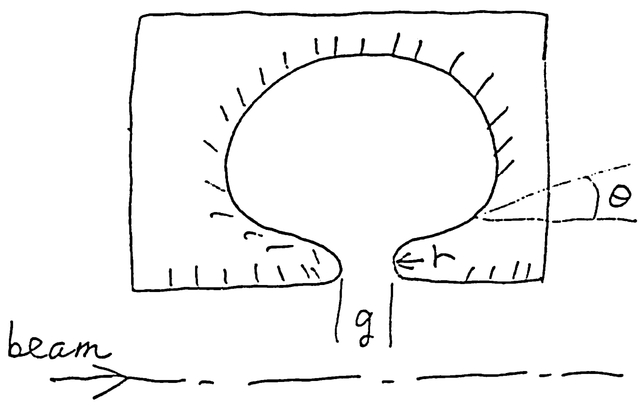
* short or long pulse, repetition
* machine の用途、維持管理体制

注意点

加速電場は平均値表示 $E_0 = \frac{1}{L}\int E\,dz$

表面最大電場 $E_p = \alpha\,E_0$

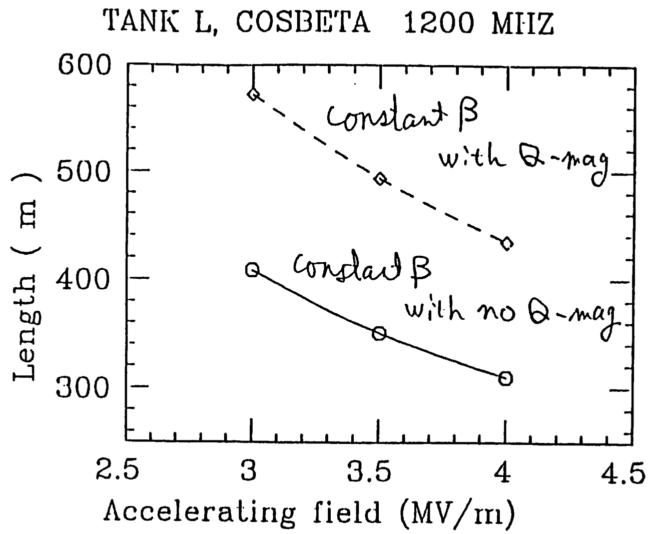
$\alpha \sim 1 - 7$

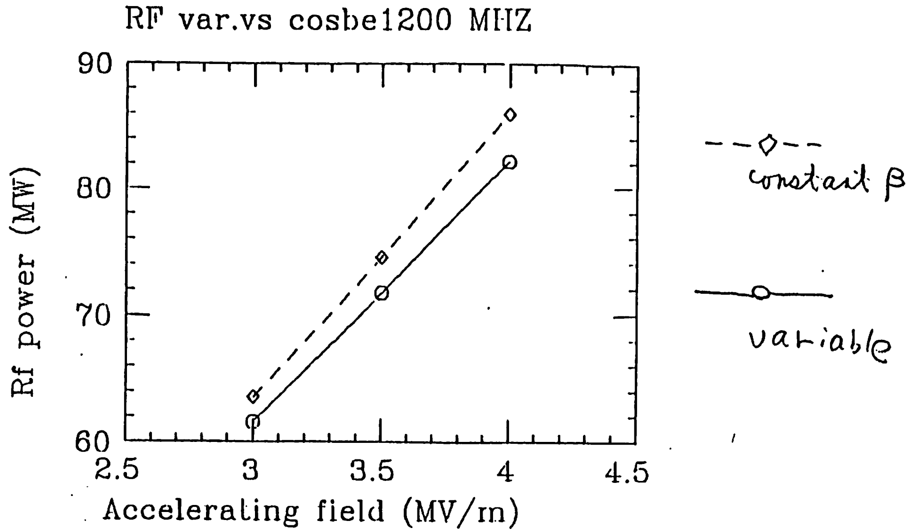


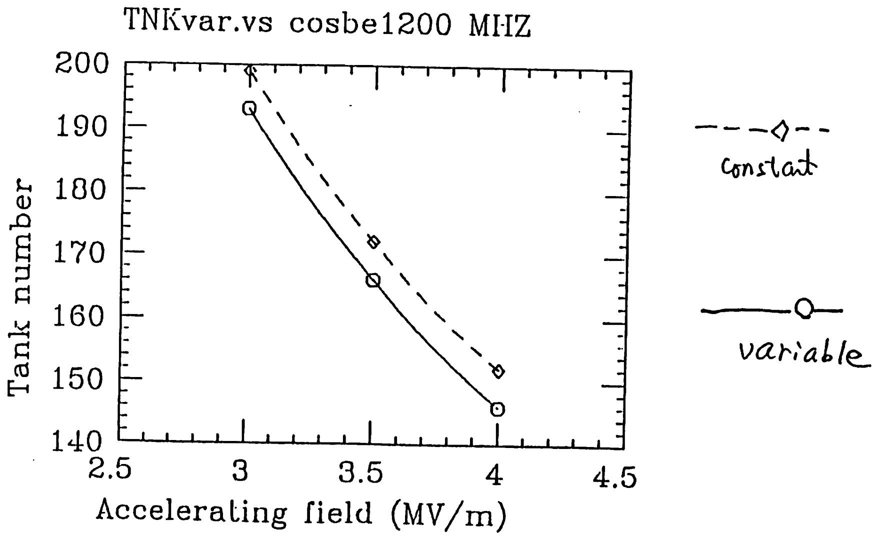
$(g, r, \theta) \Rightarrow Z, T, E_p$

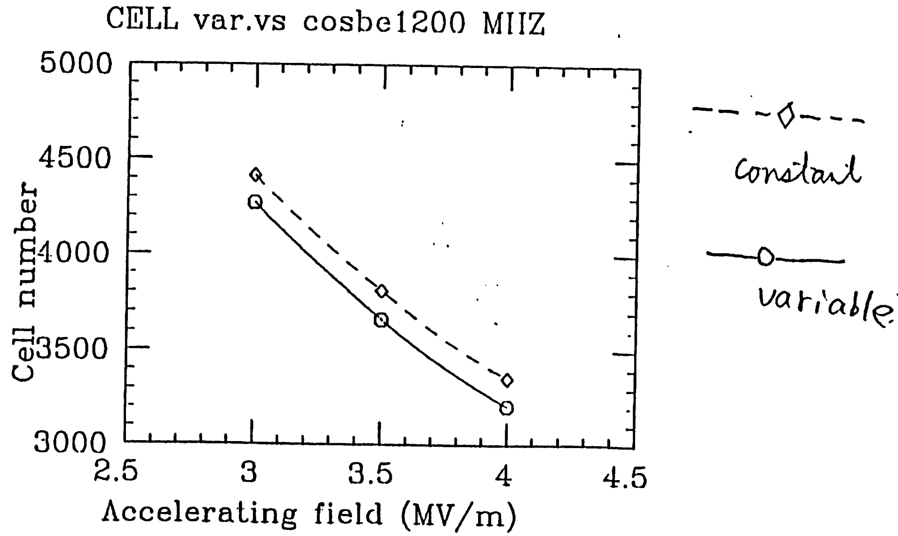
Z or ZT^2 を最適化するが

$\Delta Z + \Delta T + \Delta E \approx 0$ の範囲内か?


TANK L, COSBETA 1200 MHZ
Length (m)
600
500
400
300
2.5
3
3.5
4
4.5
Accelerating field (MV/m)
constant β with Q-mag
constant β with no Q-mag

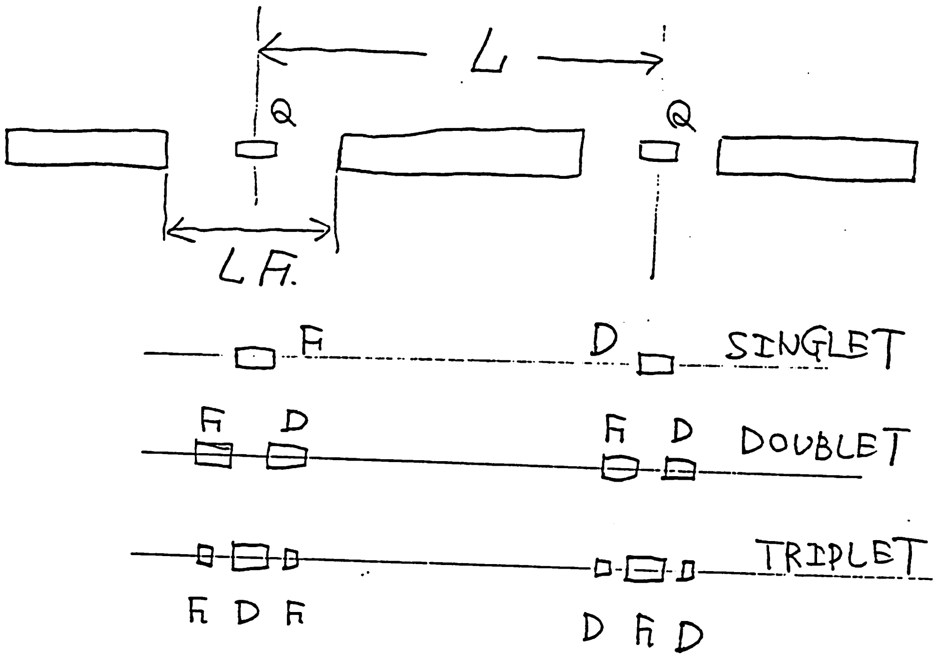

RF var.vs cosbe1200 MHZ
Rf power (MW)
90
80
70
60
2.5
3
3.5
4
4.5
Accelerating field (MV/m)
constant β
variable


TNKvar.vs cosbe1200 MHZ
Tank number
200
190
180
170
160
150
140
2.5
3
3.5
4
4.5
Accelerating field (MV/m)
constant
variable


CELL var.vs cosbe1200 MHZ
Cell number
5000
4500
4000
3500
3000
2.5
3
3.5
4
4.5
Accelerating field (MV/m)
constant
variable

CCL の focusing の方法

タンクの中へ Q-magnet を入れるのは困難
タンクとタンクのつなぎの空間を利用する

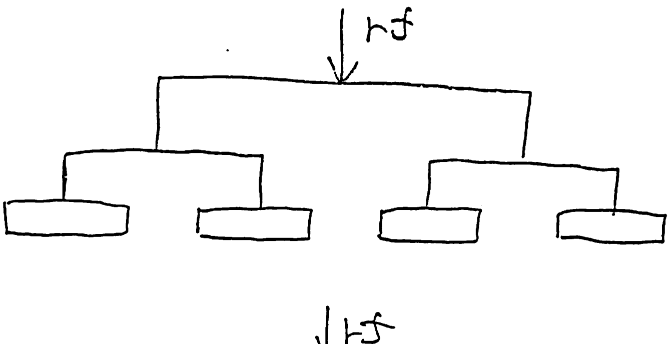


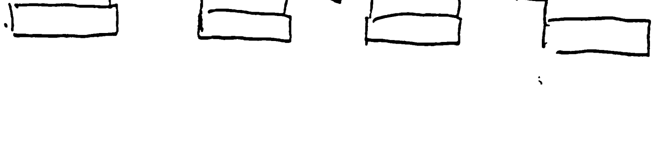
focusing period L -----> transverse acceptance

appropriate rf phase relation between two tanks

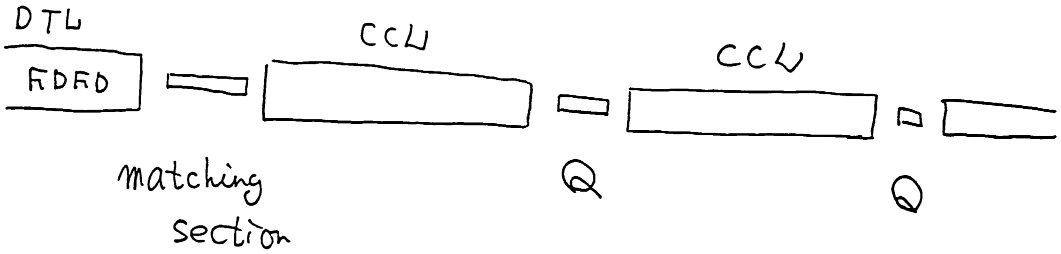
$$LF = N * \beta\lambda / 2$$

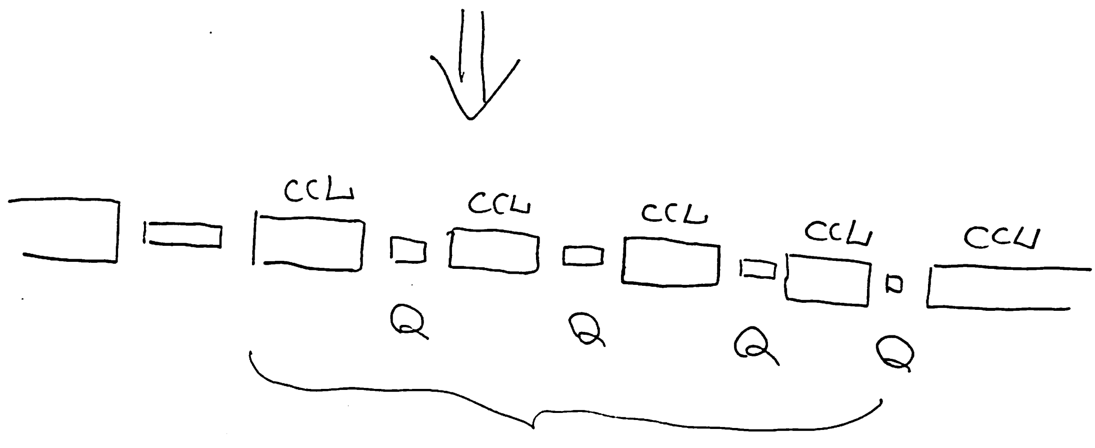
power splitter or bridge coupler
small number of klystron is desirable





DTL ⟶ CCL

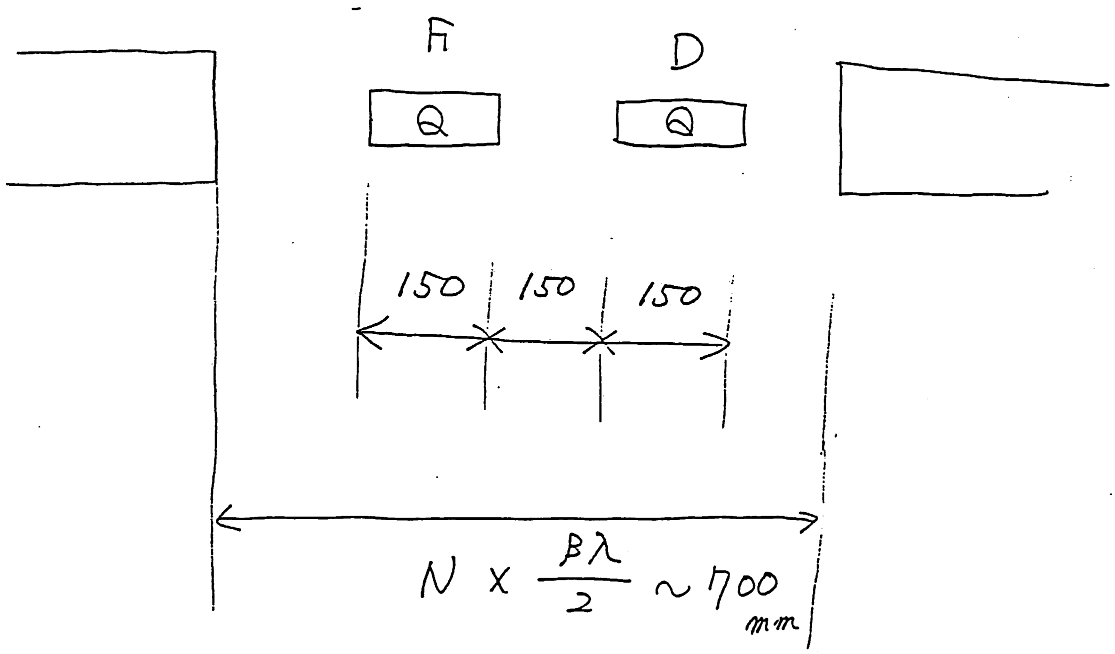




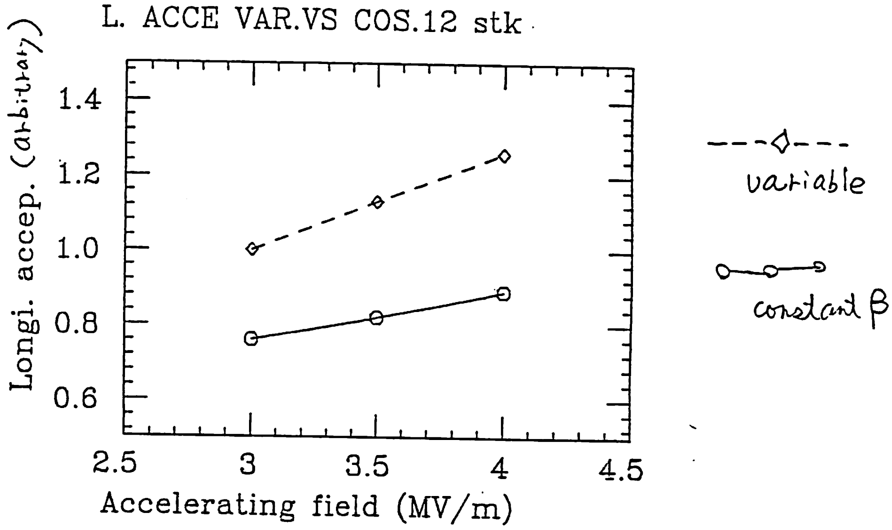
ε_t 大とできる

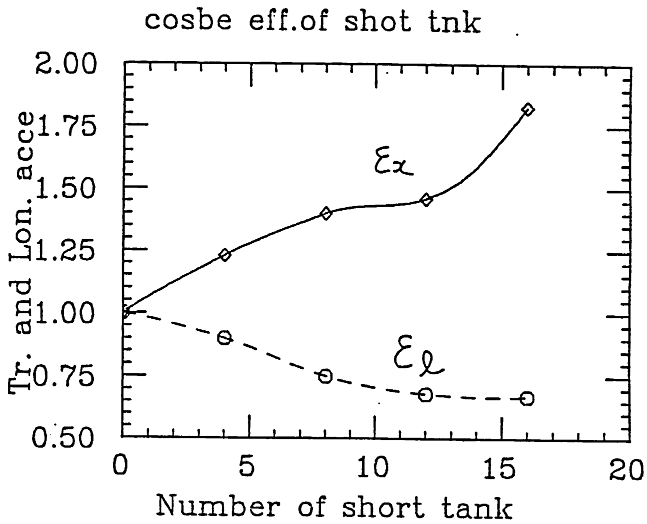
ε_ℓ は小さくなる

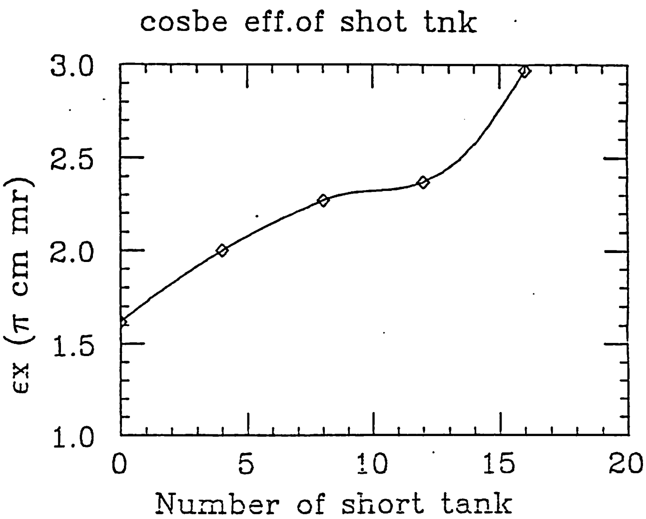
Doublet

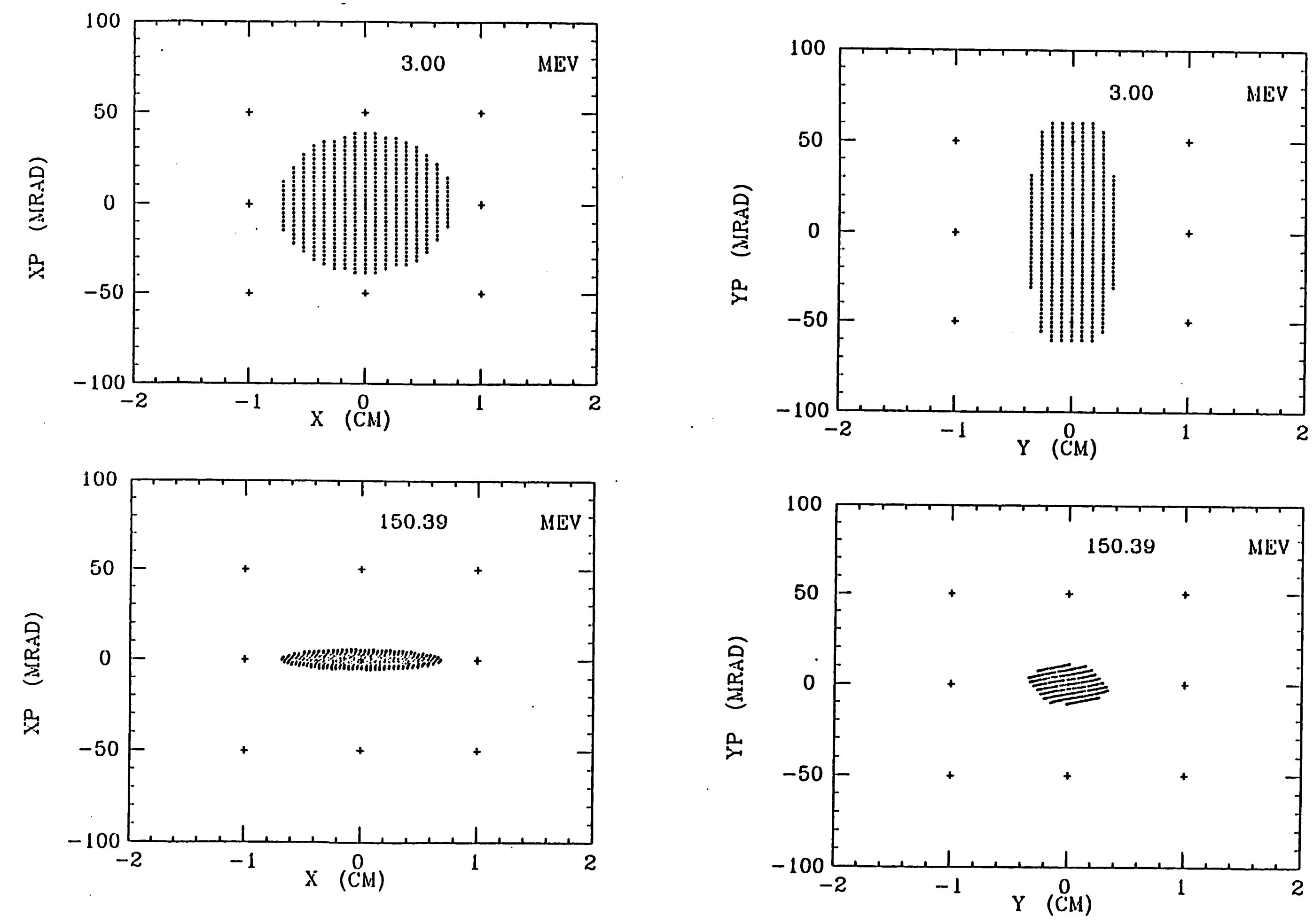


タンク No.	N
1~12	11
13~175	9
175~199	7.


L. ACCE VAR.VS COS.12 stk
Longi. accep. (arbitrary)
1.4
1.2
1.0
0.8
0.6
2.5
3
3.5
4
4.5
Accelerating field (MV/m)
variable
constant β


cosbe eff.of shot tnk
Tr. and Lon. acce
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0
5
10
15
20
Number of short tank
εx
εl


cosbe eff.of shot tnk
εx (π cm mr)
3.0
2.5
2.0
1.5
1.0
0
5
10
15
20
Number of short tank



3 MeV → 150 MeV

3 MV/m

$\varphi_s = -26°$

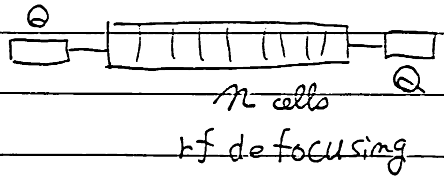
$\phi a = 15$ mm

$\mu = 76°$

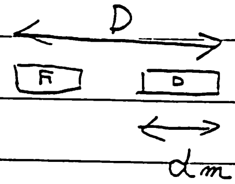
$B' = 210$ T/m

CCL Acceptance

CCL doublet fousing の（未確認の）式



◎ $\mu = \pm 1$ の limit を計算した式（であろう）



$$Q = H'^2 d_m^2 \left(D - \frac{2}{3} D_m\right)$$

$D_m = 2 d_m$ (doublet)

$D_m = 4 d_m$ (triplet)

$[H'] = \mathrm{kG/cm}$

$[d_m] = \mathrm{m}$

$$\begin{cases} Q > -0.152 \dfrac{n}{\gamma} \dfrac{E_0 T \sin\varphi}{mc^2} \\ Q < -0.076 \dfrac{n}{\gamma} \dfrac{E_0 T \sin\varphi}{mc^2} + 0.772 \dfrac{\gamma^2 \beta}{n \lambda} \end{cases}$$

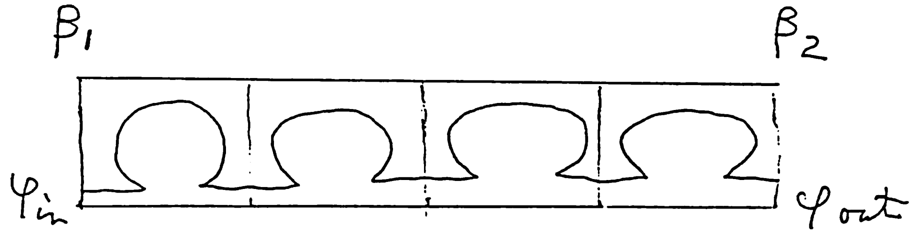
	150 MeV		1000 MeV		
	H'_{min}	H'_{max}	H'_{min}	H'_{max}	
n = 20	0.89	4.37	0.66	10.2	kG/cm
n = 40	1.26	3.19	0.94	7.2	

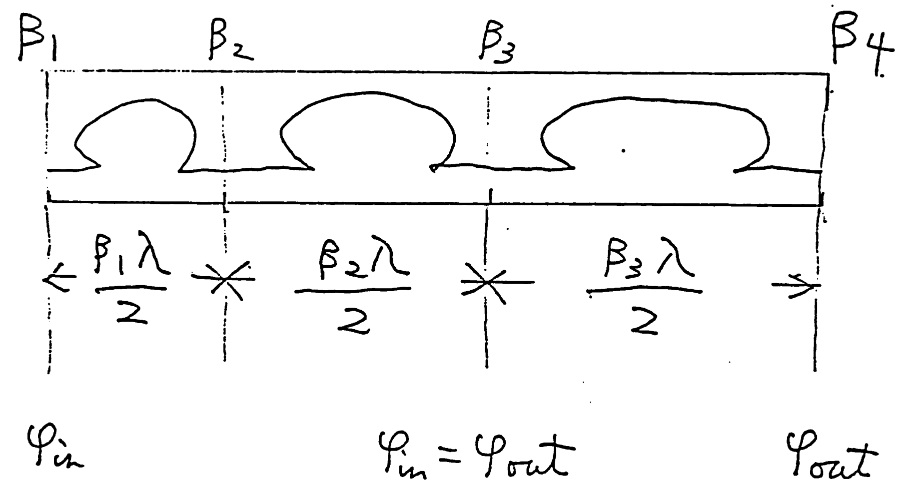
◎ 入射ビームが発散して半径がe倍になる長さを考えると

$$n^2 < -\frac{483\beta}{\pi\lambda} \frac{mc^2}{E_0 T \sin\varphi}$$

エネルギー	150 MeV	1000 MeV
n	49	152

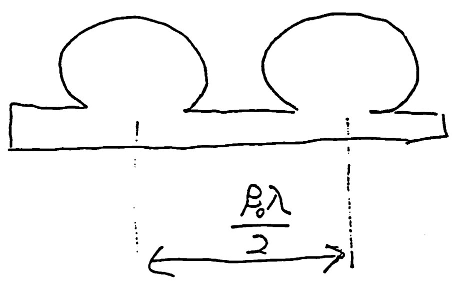
6. CONSTANT β STRUCTURE vs. VARIABLE β STRUCTURE





PHASE SLIP IN THE TANK

DECREASE OF ACCELERATING EFFICIENCY



$$\beta_i = \beta_0 + \Delta\beta$$

$$t_i = \frac{T}{2}\left(1 - \frac{\Delta\beta}{\beta_0}\right)$$

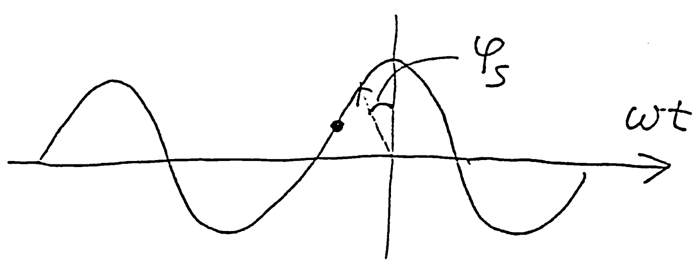
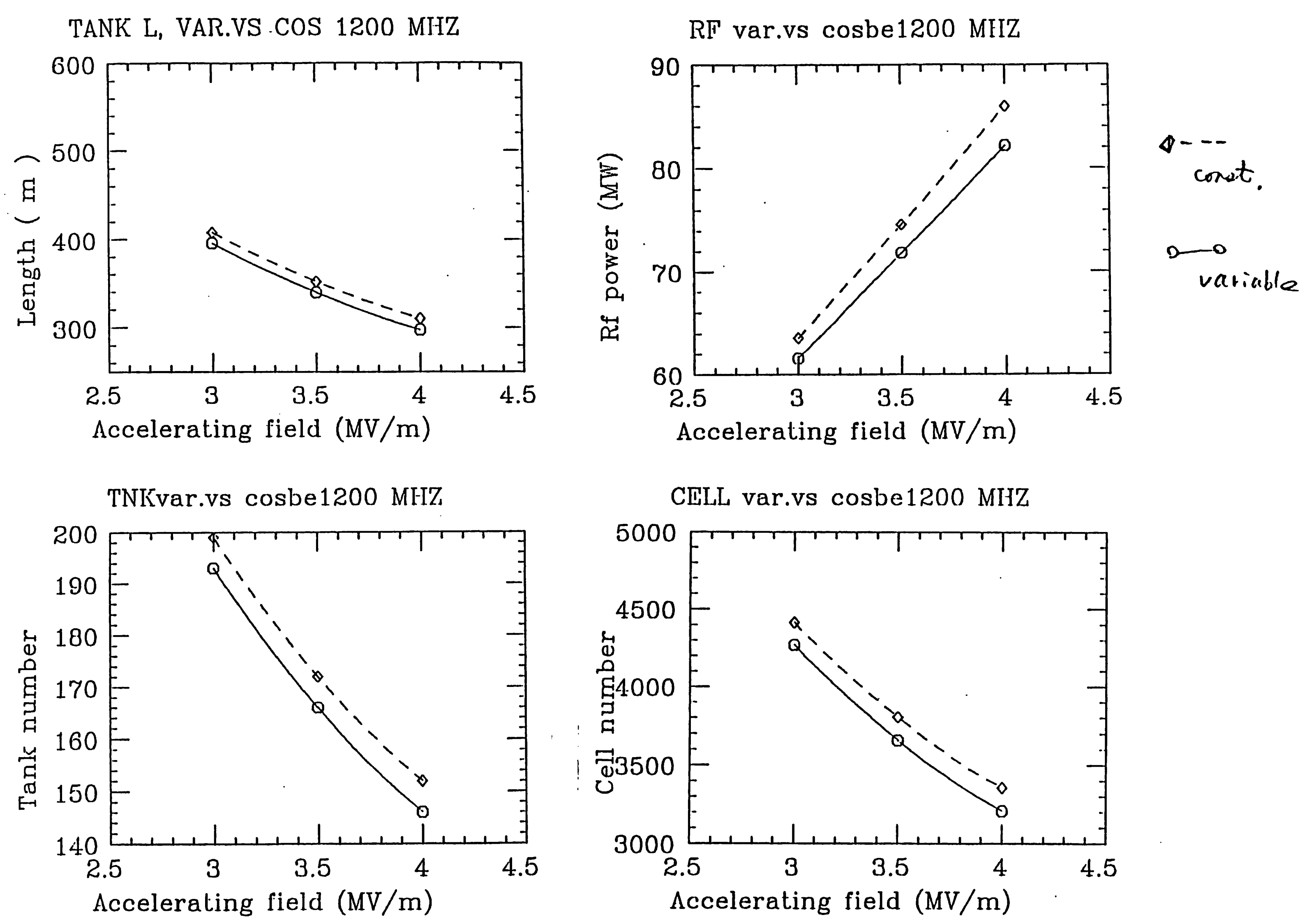


Table 1 Design of 1200 MHz SCL linac.

	constant β structure		variable β structure	
E_0 (MV/m)	3.0	4.0	3.0	4.0
TANK LENGTH (M)	408.3	310.2	396.0	297.4
WHOLE LENGTH (m)	572.5	434.8	553.4	416.1
TANK NUMBER	199	152	193	146
CELL NUMBER	4414	3354	4270	3206
RF POWER (MW)	63.6	86.0	61.6	82.2
ϕ_1 (degree)	-41.8	-45.6	-30	-30
ϕ_2 (degree)	-52.6	-58.2	-30	-30
average ϕ (deg)	-31.7	-32.3	-30	-30

注 1. SCL の最初の 12 tanks は short tank.
注 2. WHOLE LENGTH は Q - magnet section を含むので、変わりうる。
注 3. RF power は、shunt impedance Z を 1/1.3 倍して計算している。
注 4. ϕ_1 は short tank 内の最小 stable phase.
注 5. ϕ_2 は long tank 内の最小 stable phase.

const β vs variable β.



6-3

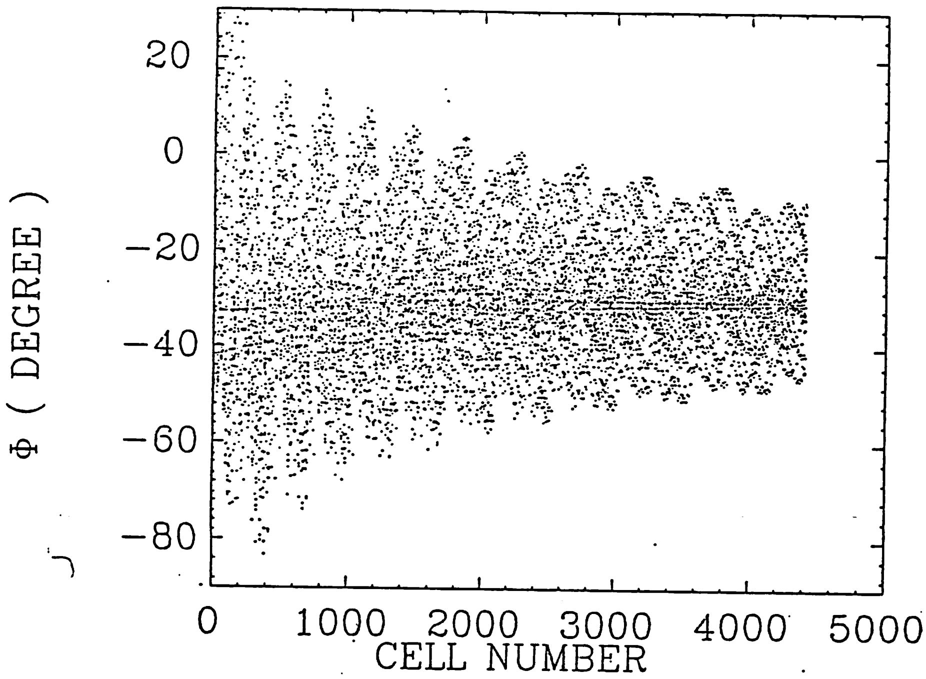


Fig.11 Longitudinal oscillation in the constant β structure.

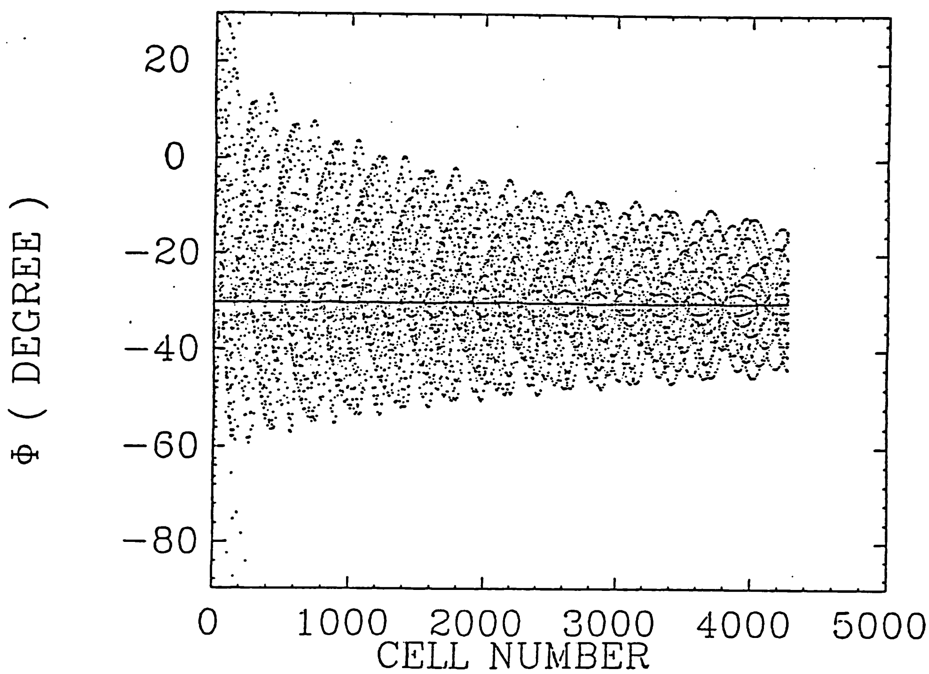


Fig.12 Longitudinal oscillation in the variable β structure.

6-5

NTK	N1	N2	NC	WIN MeV	BETAIN	CLENG cm	TANKLENG cm	RF MWx1.3	ZS MOHM/m	WGAIN MeV	PHI1 deg	PHI2 deg
1	1	20	20	150.000	0.5066	6.3298	126.5959	0.486	54.133	3.3695	-30.00	-45.59
2	21	40	20	153.369	0.5111	6.3860	127.7203	0.487	54.595	3.4064	-30.00	-45.33
3	41	60	20	156.776	0.5156	6.4418	128.8367	0.487	55.053	3.4431	-30.00	-45.08
4	61	80	20	160.219	0.5200	6.4972	129.9449	0.487	55.508	3.4796	-30.00	-44.84
5	81	100	20	163.698	0.5244	6.5523	131.0450	0.487	55.959	3.5159	-30.00	-44.60
6	101	120	20	167.214	0.5288	6.6068	132.1369	0.487	56.407	3.5520	-30.00	-44.36
7	121	140	20	170.766	0.5331	6.6610	133.2204	0.487	56.852	3.5879	-30.00	-44.13
8	141	160	20	174.354	0.5374	6.7148	134.2957	0.488	57.293	3.6236	-30.00	-43.90
9	161	180	20	177.978	0.5417	6.7681	135.3625	0.488	57.731	3.6590	-30.00	-43.68
10	181	200	20	181.637	0.5460	6.8211	136.4210	0.488	58.166	3.6943	-30.00	-43.46
11	201	220	20	185.331	0.5502	6.8736	137.4711	0.488	58.596	3.7293	-30.00	-43.24
12	221	240	20	189.060	0.5543	6.9256	138.5127	0.488	59.024	3.7641	-30.00	-43.03
13	241	270	30	192.824	0.5585	6.9773	209.3187	0.732	59.448	5.2737	-30.00	-58.20
14	271	300	30	198.098	0.5641	7.0482	211.4452	0.733	60.030	5.3514	-30.00	-57.61
15	301	330	30	203.450	0.5698	7.1184	213.5518	0.734	60.547	5.4257	-30.00	-57.02
16	331	358	28	208.875	0.5753	7.1879	201.2613	0.686	61.029	5.2234	-30.00	-53.17
17	359	386	28	214.099	0.5806	7.2533	203.0917	0.687	61.483	5.2853	-30.00	-52.68
18	387	414	28	219.384	0.5857	7.3179	204.9024	0.688	61.932	5.3465	-30.00	-52.21
19	415	442	28	224.730	0.5909	7.3819	206.6932	0.689	62.376	5.4071	-30.00	-51.75
20	443	470	28	230.138	0.5959	7.4451	208.4640	0.690	62.815	5.4671	-30.00	-51.29
21	471	498	28	235.605	0.6009	7.5077	210.2148	0.691	63.249	5.5263	-30.00	-50.85
22	499	526	28	241.131	0.6059	7.5695	211.9454	0.692	63.678	5.5849	-30.00	-50.42
23	527	554	28	246.716	0.6108	7.6306	213.6559	0.693	64.102	5.6428	-30.00	-50.00
24	555	582	28	252.359	0.6156	7.6909	215.3463	0.694	64.521	5.7001	-30.00	-49.58
25	583	608	26	258.059	0.6204	7.7506	201.5153	0.645	64.935	5.4119	-30.00	-46.60
26	609	634	26	263.471	0.6248	7.8060	202.9566	0.646	65.320	5.4597	-30.00	-46.28
27	635	660	26	268.930	0.6292	7.8608	204.3810	0.647	65.700	5.5069	-30.00	-45.96
28	661	686	26	274.437	0.6336	7.9149	205.7885	0.648	66.076	5.5535	-30.00	-45.65
29	687	712	26	279.991	0.6378	7.9684	207.1791	0.649	66.448	5.5996	-30.00	-45.35
30	713	738	26	285.590	0.6421	8.0213	208.5529	0.649	66.814	5.6451	-30.00	-45.05
31	739	764	26	291.235	0.6462	8.0735	209.9100	0.650	67.177	5.6901	-30.00	-44.76
32	765	790	26	296.926	0.6504	8.1250	211.2505	0.651	67.535	5.7346	-30.00	-44.48
33	791	816	26	302.660	0.6545	8.1759	212.5745	0.651	67.888	5.7785	-30.00	-44.20
34	817	842	26	308.439	0.6585	8.2262	213.8820	0.652	68.237	5.8219	-30.00	-43.93
35	843	868	26	314.261	0.6625	8.2759	215.1733	0.653	68.582	5.8647	-30.00	-43.67
36	869	894	26	320.125	0.6664	8.3249	216.4483	0.653	68.923	5.9070	-30.00	-43.41
37	895	918	24	326.032	0.6703	8.3734	200.9605	0.604	69.259	5.5366	-30.00	-41.23
38	919	942	24	331.569	0.6738	8.4179	202.0291	0.604	69.568	5.5714	-30.00	-41.03
39	943	966	24	337.140	0.6773	8.4619	203.0848	0.605	69.873	5.6058	-30.00	-40.83
40	967	990	24	342.746	0.6808	8.5053	204.1278	0.605	70.175	5.6397	-30.00	-40.64
41	991	1014	24	348.386	0.6843	8.5483	205.1581	0.606	70.473	5.6732	-30.00	-40.46
42	1015	1038	24	354.059	0.6877	8.5907	206.1758	0.606	70.767	5.7063	-30.00	-40.28
43	1039	1062	24	359.765	0.6910	8.6325	207.1811	0.606	71.058	5.7391	-30.00	-40.10
44	1063	1086	24	365.504	0.6943	8.6739	208.1741	0.607	71.345	5.7714	-30.00	-39.92
45	1087	1110	24	371.276	0.6976	8.7148	209.1549	0.607	71.629	5.8033	-30.00	-39.75
46	1111	1134	24	377.079	0.7008	8.7552	210.1236	0.608	71.909	5.8349	-30.00	-39.58
47	1135	1158	24	382.914	0.7040	8.7950	211.0803	0.608	72.186	5.8660	-30.00	-39.42
48	1159	1182	24	388.780	0.7072	8.8344	212.0252	0.609	72.459	5.8968	-30.00	-39.26
49	1183	1206	24	394.677	0.7103	8.8733	212.9583	0.609	72.729	5.9271	-30.00	-39.10
50	1207	1230	24	400.604	0.7134	8.9117	213.8798	0.609	72.992	5.9568	-30.00	-38.95
51	1231	1254	24	406.561	0.7164	8.9496	214.7898	0.610	73.237	5.9841	-30.00	-38.79
52	1255	1278	24	412.545	0.7194	8.98[illegible]	215.6880	0.611	73.478	6.0110	[illegible]0.00	-38.64
53	1279	1302	24	418.556	0.7224	9.0239	216.5747	0.611	73.716	6.0376	-30.00	-38.49
54	1303	1326	24	424.593	0.7253	9.0604	217.4500	0.612	73.951	6.0638	-30.00	-38.35

6-6

LINOUT SUBROUTINE NO. 1   DYNAMICAL PARAMETERS

TANK NO. 1   TANK LENGTH 31015.91988 CENTIMETERS   3354 CELLS   POWER= 85.963 MW

CELL NUMBER	KINETIC ENERGY	BETA	LENGTH	T	TP	S	SP	STABLE PHASE	EZERO MV/M	Z	TOTAL LENGTH	RFPOW
NITIAL	150.0000	0.5066						0.0	0.0			0.0
5	150.8916	0.5078	6.3298	0.8150	0.0540	0.4540	0.0540	-30.8517	4.0000	54.133	31.649	0.094
10	151.7661	0.5090	6.3298	0.8150	0.0540	0.4540	0.0540	-33.8009	4.0000	54.133	63.298	0.187
15	152.6011	0.5101	6.3298	0.8150	0.0540	0.4540	0.0540	-38.7607	4.0000	54.133	94.947	0.281
20	153.3695	0.5111	6.3298	0.8150	0.0540	0.4540	0.0540	-45.5893	4.0000	54.133	126.596	0.374
25	154.2698	0.5123	6.3860	0.8158	0.0538	0.4533	0.0540	-30.8372	4.0000	54.595	158.526	0.468
30	155.1533	0.5135	6.3860	0.8158	0.0538	0.4533	0.0540	-33.7368	4.0000	54.595	190.456	0.561
35	155.9976	0.5146	6.3860	0.8158	0.0538	0.4533	0.0540	-38.6147	4.0000	54.595	222.386	0.655
40	156.7758	0.5156	6.3860	0.8158	0.0538	0.4533	0.0540	-45.3339	4.0000	54.595	254.316	0.748
45	157.6849	0.5168	6.4418	0.8165	0.0535	0.4526	0.0540	-30.8230	4.0000	55.053	286.525	0.842
50	158.5773	0.5179	6.4418	0.8165	0.0535	0.4526	0.0540	-33.6740	4.0000	55.053	318.735	0.936
55	159.4308	0.5190	6.4418	0.8165	0.0535	0.4526	0.0540	-38.4715	4.0000	55.053	350.944	1.029
60	160.2189	0.5200	6.4418	0.8165	0.0535	0.4526	0.0540	-45.0833	4.0000	55.053	383.153	1.123
65	161.1367	0.5212	6.4972	0.8173	0.0533	0.4520	0.0540	-30.8091	4.0000	55.508	415.639	1.217
70	162.0378	0.5223	6.4972	0.8173	0.0533	0.4520	0.0540	-33.6124	4.0000	55.508	448.125	1.310
75	162.9005	0.5234	6.4972	0.8173	0.0533	0.4520	0.0540	-38.3311	4.0000	55.508	480.612	1.404
80	163.6985	0.5244	6.4972	0.8173	0.0533	0.4520	0.0540	-44.8375	4.0000	55.508	513.098	1.497
85	164.6249	0.5256	6.5523	0.8180	0.0531	0.4513	0.0540	-30.7955	4.0000	55.959	545.859	1.591
90	165.5348	0.5267	6.5523	0.8180	0.0531	0.4513	0.0540	-33.5521	4.0000	55.959	578.620	1.685
95	166.4067	0.5278	6.5523	0.8180	0.0531	0.4513	0.0540	-38.1935	4.0000	55.959	611.382	1.778
100	167.2144	0.5288	6.5523	0.8180	0.0531	0.4513	0.0540	-44.5963	4.0000	55.959	644.143	1.872
105	168.1494	0.5300	6.6068	0.8187	0.0529	0.4506	0.0540	-30.7822	4.0000	56.407	677.177	1.966
110	169.0680	0.5311	6.6068	0.8187	0.0529	0.4506	0.0540	-33.4929	4.0000	56.407	710.211	2.060
115	169.9489	0.5322	6.6068	0.8187	0.0529	0.4506	0.0540	-38.0586	4.0000	56.407	743.245	2.153
120	170.7664	0.5331	6.6068	0.8187	0.0529	0.4506	0.0540	-44.3597	4.0000	56.407	776.280	2.247
125	171.7099	0.5343	6.6610	0.8195	0.0527	0.4500	0.0540	-30.7691	4.0000	56.852	809.585	2.341
130	172.6373	0.5354	6.6610	0.8195	0.0527	0.4500	0.0540	-33.4350	4.0000	56.852	842.890	2.434
135	173.5272	0.5365	6.6610	0.8195	0.0527	0.4500	0.0540	-37.9262	4.0000	56.852	876.195	2.528
140	174.3543	0.5374	6.6610	0.8195	0.0527	0.4500	0.0540	-44.1275	4.0000	56.852	909.500	2.622
145	175.3063	0.5386	6.7148	0.8202	0.0524	0.4493	0.0540	-30.7563	4.0000	57.293	943.074	2.716
150	176.2422	0.5397	6.7148	0.8202	0.0524	0.4493	0.0540	-33.3781	4.0000	57.293	976.648	2.809
155	177.1411	0.5407	6.7148	0.8202	0.0524	0.4493	0.0540	-37.7964	4.0000	57.293	1010.222	2.903
160	177.9778	0.5417	6.7148	0.8202	0.0524	0.4493	0.0540	-43.8997	4.0000	57.293	1043.796	2.997
165	178.9383	0.5428	6.7681	0.8209	0.0522	0.4487	0.0540	-30.7438	4.0000	57.731	1077.636	3.091
170	179.8828	0.5439	6.7681	0.8209	0.0522	0.4487	0.0540	-33.3224	4.0000	57.731	1111.477	3.185
175	180.7906	0.5450	6.7681	0.8209	0.0522	0.4487	0.0540	-37.6691	4.0000	57.731	1145.318	3.278
180	181.6368	0.5460	6.7681	0.8209	0.0522	0.4487	0.0540	-43.6760	4.0000	57.731	1179.158	3.372
185	182.6057	0.5471	6.8211	0.8216	0.0520	0.4480	0.0540	-30.7315	4.0000	58.166	1213.264	3.466
190	183.5587	0.5482	6.8211	0.8216	0.0520	0.4480	0.0540	-33.2677	4.0000	58.166	1247.369	3.560
195	184.4754	0.5492	6.8211	0.8216	0.0520	0.4480	0.0540	-37.5442	4.0000	58.166	1281.474	3.654
200	185.3311	0.5502	6.8211	0.8216	0.0520	0.4480	0.0540	-43.4566	4.0000	58.166	1315.579	3.747
205	186.3083	0.5513	6.8736	0.8223	0.0518	0.4474	0.0540	-30.7194	4.0000	58.596	1349.947	3.841
210	187.2698	0.5523	6.8736	0.8223	0.0518	0.4474	0.0540	-33.2142	4.0000	58.596	1384.315	3.935
215	188.1952	0.5534	6.8736	0.8223	0.0518	0.4474	0.0540	-37.4217	4.0000	58.596	1418.683	4.029
220	189.0604	0.5543	6.8736	0.8223	0.0518	0.4474	0.0540	-43.2412	4.0000	58.596	1453.050	4.123
225	190.0459	0.5554	6.9256	0.8230	0.0516	0.4468	0.0540	-30.7076	4.0000	59.024	1487.679	4.217
230	191.0157	0.5565	6.9256	0.8230	0.0516	0.4468	0.0540	-33.1616	4.0000	59.024	1522.307	4.310
235	191.9499	0.5575	6.9256	0.8230	0.0516	0.4468	0.0540	-37.3015	4.0000	59.024	1556.935	4.404
240	192.8245	0.5585	6.9256	0.8230	0.0516	0.4468	0.0540	-43.0297	4.0000	59.024	1591.563	4.498
245	193.8182	0.5595	6.9773	0.8237	0.0514	0.4461	0.0540	-30.6959	4.0000	59.448	1626.450	4.592
250	194.7964	0.5606	6.9773	0.8237	0.0514	0.4461	0.0540	-33.1100	4.0000	59.448	1661.336	4.686
255	195.7393	0.5616	6.9773	0.8237	0.0514	0.4461	0.0540	-37.1835	4.0000	59.448	1696.222	4.780

7. 150 MeV DTL

3 MeV ---> 150 MeV, 400 MHz
20 mA, 400 μs beam, 50 pps

TANK DIAMETER	47 - 43 cm		
DRIFT TUBE DIAMETER	9 - 7 cm		
BEAM HOLE RADIUS	0.75 cm		
CELL LENGTH	6.03 - 38 cm		
ACCEL. FIELD	3	2.4	MV/m
LENGTH	72.0	89.9	m
RF POWER (X 1.3)	9.8	7.8	MW
NO. OF CELLS	302	377	
STABLE PHASE	-26	-26	
$\varepsilon_{x,n}$	2.1		π cm mr
$\varepsilon_{y,n}$	1.9		

POST-STABILIZED DTL

PERMANENT QUADRUPOLE MAGNET

8. RFQ

$$\Delta W = e\ E_0 T \cos\phi$$

$$E_0{}' = 2AV/(\beta\lambda),\quad T = \pi/4$$

$$A = (m^2 - 1)/(m^2 I_0(ka) + I_0(mka))$$

$$k = 2\pi/(\beta\lambda)$$

E_0 は平均加速電場、V は電極間電圧、m は電極の modulation の大きさ、a は最小半径、I_0 は変形ベッセル関数である。

高周波的に独立な一つのタンク内では電極間電圧は一定であるので、m が一定の時は、ほぼ β に反比例して平均加速電場は減少する。逆に、平均加速電場 1 MV/m を得ようとすれば、Fig.1 に示す様に電極間電圧はエネルギーとともに変わる。同じ事であるが、750 keV における加速効率（ E_0 / V ）を基準にすると、より高いエネルギーの時の加速効率は Fig.2 に示す様に減少する。ここでは、m = 2, a = 0.004 と仮定している。

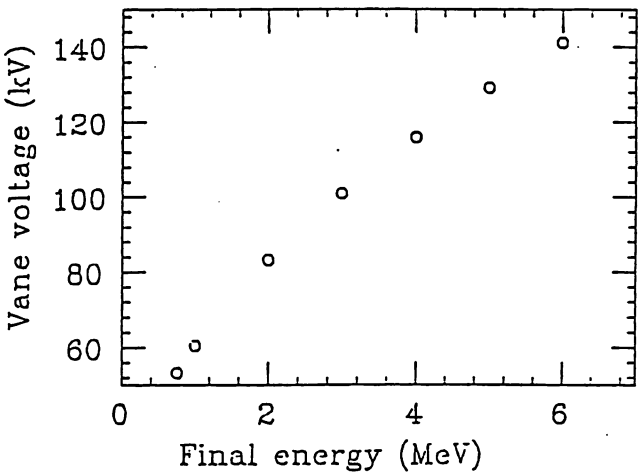


Fig.1 Variation of vane voltage vs. final beam energy.

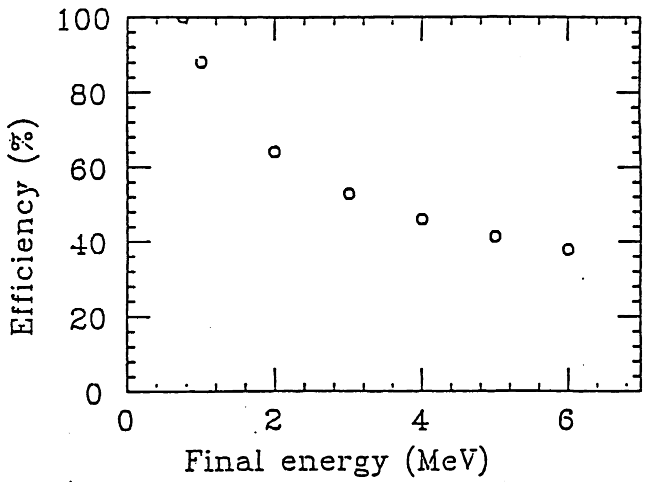


Fig.2 Decrease of acceleration efficiency vs. final beam energy.

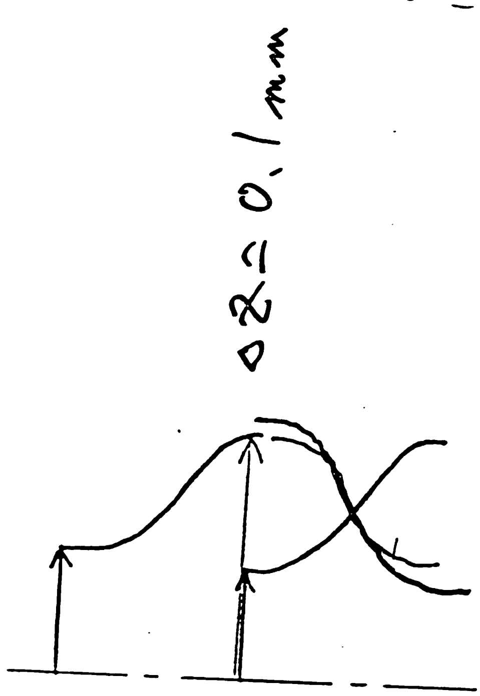
Table 4 3 MeV RFQ (400 MHz)

最小半径	2.5	2.0	mm
final enrgy	3.0	3.0	MeV
length	2.765	3.514	m
cell number	282	330	
vane voltage	109	95	kV
ε x,n(90%)	0.1	0.07	π cm mr
rf power	650	430	kW

QKEK vs PARMTEQ

input parameters

QKEK	PARMTEQ
$E(j)$	$B(j) = B(j, m_j, a_j)$
$\phi(j)$	$\phi(j)$
m_0	$m(j)$
a_0	V
output	
$B(j)$	$E(j)$
$a(j) - 1$	$a(j)$
$m(j) - 1$	
V	



PARMTEQ

$$\left.\begin{matrix} B(j) \\ m(j) \\ V \end{matrix}\right\} \Rightarrow a(j)$$

$$\left.\begin{matrix} a(j) \\ m(j) \\ V \end{matrix}\right\} \Rightarrow E(j)$$

QKEK

$$\left.\begin{matrix} m_0 \\ a_0 \\ E_z(L) \end{matrix}\right\} \Rightarrow V$$

$$\left.\begin{matrix} V \\ E(j) \\ a(j+1) \end{matrix}\right\} \Rightarrow m(j) \Rightarrow B(j)$$

$a_1 = m_1 a_0$

$a_2 = a_1 / m_2$

⋮