

## Injection energy and equipartitioning in DTL

Emittance growth sometimes occurs in the low energy section of the Alvarez linac. To avoid the emittance growth is one of the main problems in designing linear accelerators. The idea of equipartitioning between the degrees of freedom is useful to understand the effects(ref). The unnormalized transverse and longitudinal rms emittances are written for the matched beams as

$$\begin{aligned} \mathcal{E}_t &= \frac{\alpha^t a^2}{N \beta \lambda} \\ \mathcal{E}_l &= \frac{\alpha^l b^2}{N \beta \lambda} \end{aligned} \quad (1) \quad (N=2 \text{ for DTL})$$

where  $\alpha^t$  and  $\alpha^l$  are phase advances over an accelerator system period,  $a$  is the average transverse rms beam radius and  $2b$  is the physical rms bunch length. If we require equal average energy in each of the coupled degrees of freedom, it is found

$$\frac{\mathcal{E}_l}{\mathcal{E}_t} = \frac{\alpha^t}{\alpha^l} = \frac{b}{a} \quad (2)$$

Systems satisfying the above two equations simultaneously will be both matched and equipartitioned. Phase advances are given as follows.

$$\alpha^t = \cos^{-1} \left( \cos \alpha_0^t + \frac{30 I \lambda^3}{W_0} \frac{(3b-a)}{a^2 b^2} \right) \quad (3)$$

$$\alpha^l = 2 \cos^{-1} \left( \cos \alpha_0^l + \frac{15 I \lambda^3}{W_0 a b^2} \right) \quad (4)$$

$$\alpha_0^t = \cos^{-1} \left( 1 - \left( \frac{1}{2} - \frac{\Lambda}{3} \right) \Lambda^2 Q_0^4 - \frac{2\pi E_0 T \lambda \sin \phi_s}{W_0 \beta} \right) \quad (5)$$

$$\alpha_0^l = \cos^{-1} \left( 1 + \frac{\pi E_0 T \lambda \sin \phi_s}{W_0 \beta} \right) \quad (6)$$

$$B' = \frac{W_0}{C \beta \lambda^2} Q_0^2 \quad (7)$$

where  $I$  is average current,  $W_0$  is rest energy,  $\Lambda$  is the quad-filling factor,  $E_0$  is the average accelerating gradient,  $T$  is transit time factor,  $\lambda$  is wavelength,  $\phi_s$  is synchronous phase angle,  $B'$  is the quad gradient. Space charge parameters are defined as the ratio of space charge to external forces,

$$\mu_t = 1 - \left( \frac{\alpha^t}{\alpha_0^t} \right)^2, \quad \mu_l = 1 - \left( \frac{\alpha^l}{\alpha_0^l} \right)^2 \quad (8)$$

The ratio of longitudinal matched emittance to transverse one is derived from equation (1),

$$e_L / e_T = \text{sigL} * b * b / (\text{sigT} * a * a).$$

$$\left( \begin{array}{l} e_L \equiv \epsilon_L, \quad \text{sigL} \equiv \alpha^L \\ e_T \equiv \epsilon_t, \quad \text{sigT} \equiv \alpha^t \end{array} \right)$$

In this note, the achieve-ratio is defined as follows,

$$\text{achieve ratio} = \frac{\epsilon_L}{\epsilon_t} / \frac{\alpha^t}{\alpha^L}$$

Therefore, achieve-ratio = 1 means an acceleration of both matched and equipartitioned beam.

The zero-current transverse phase advance is assumed to be 60 degrees and the normalized - 90% beam emittance is assumed 0.28 pi cm mrad. The relation between the 90% emittance and the rms emittance is assumed that E(90)=4\*E(rms). Beam dynamics parameters are calculated for two kinds of injection energy, two kinds of accelerating field and the beam current of 20 mA. The results are shown in Fig.1 and the matched and equipartitioned beams are summarized in Tabel 1.

Figure 1 shows there is one solution of matched and equipartitioned bunch length when the input beam emittance is given. In the case of 2 MeV injection, the bunch length of 17.4 degrees is desirable and the bunch length of 15.5 degrees are desirable for the 3 MeV injection. If the accelerating field decreases to 2 MV/m, the optimized bunch length about 20 degrees seems too large to accept and accelerate the beam stably, because the bunch length into the drift-tube linac has a large influence on the behavior of the beam loss in the high-beta section of the linac.

If we use an RFQ linac for the preinjector, we can expect an narrow bunch length and if we use a buncher cavity between RFQ and DTL, we can control the bunch length to achieve the optimized bunch length.

Table 1 Matched and equipartitioned beam parameters for the 432 MHz linac.

Injection energy	2	3	MeV
beta	0.0652	0.080	
T	0.752	0.790	
E0 = 3 MV/m, I=20 mA, stable phase angle=-30			
Sig0L	16.3	15.1	
SigL	22.9	21.4	
SigT	45.4	46.6	
SigL/Sig0L	0.702	0.709	
muL	0.507	0.498	
SigT/Sig0T	0.757	0.777	
muT	0.427	0.397	
b	17.4	15.5	degrees
a	0.11	0.11	cm
eT(n,90%) (1)	0.28	0.28	pi cm mrad

SigT/SigL	1.98	2.17
eL / eT	1.99	2.17
b/a	1.99	2.17
achieve-ratio	1.00	1.00

E0 = 2 MV/m, I=20 mA, stable phase angle=-30

Sig0L	13.3	12.3
SigL	19.3	17.9
SigT	48.2	49.1
SigL/Sig0L	0.726	0.728
muL	0.474	0.471
SigT/Sig0T	0.803	0.818
muT	0.355	0.330
b	22.0	19.5 degrees
a	0.11	0.11 cm
eT(n,90%) (1)	0.29	0.30 pi cm mrad
SigT/SigL	2.50	2.74
eL / eT	2.52	2.73
b/a	2.51	2.73
achieve-ratio	1.01	1.00

We show the results of the same calculation applied to the conventional linac of 750 keV-injection-energy and 201 MHz-operating frequency in Table 2 and Fig.2. Here, the beam current is assumed 100 mA and 20 mA. The normalized-90% beam emittance is assumed 0.4 pi cm mrad.

Table 2 Matched and equipartitioned beam parameters for the 201 MHz - 750 keV - injection linac.

stable phase angle=-30

	100	100	20	mA
E0	1.5	2.0	1.5	MV/m
Sig0L	19.8	22.9	19.8	
SigL	9.82	11.0	22.5	
SigT	16.8	15.2	36.3	
SigL/Sig0L	0.248	0.240	0.568	
muL	0.939	0.942	0.677	
SigT/Sig0T	0.280	0.253	0.605	
muT	0.922	0.936	0.634	
b	33.0	27.9	21.2	degrees
a	0.320	0.335	0.217	cm
eT(n,90%)	0.40	0.40	0.4	picm mrad
SigT/SigL	1.71	1.38	1.61	
eL / eT	1.71	1.38	1.62	
b/a	1.71	1.38	1.62	
achieve-ratio	1.0	1.0	1.0	

The space charge parameters for the beam current of 100 mA are very large, which appears in Fig.2 in the form of the steep gradient of the line. In this case the longitudinal tuning of the beam to achieve the matched and equipartitioned condition is very sensitive and difficult. Moreover, the rms half bunch length of 30 degrees seems too large to accept the beam though the matched

and equipartitioned condition is satisfied.

One should note that the estimation mentioned above strongly depends on the values of the transverse and longitudinal emittances of the beam.

It should be also noted that the linac with equipartitioning situation is desirable or not when the space charge force is not so strong. In other words, how much deviation from the matched and equipartitioned condition is allowable.

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ref. R.A.Jameson,"Equipartitioning in linear acceleartors",1981 Linear Acceleartor Conference,p.125.

$\omega_p^2 = 60, 432 \text{ MHz}$   
 $I = 20 \text{ mA}$   
 $E_{N,90} = 0.28 \text{ MV/m}$

# ACHIEVE-RATIO 432

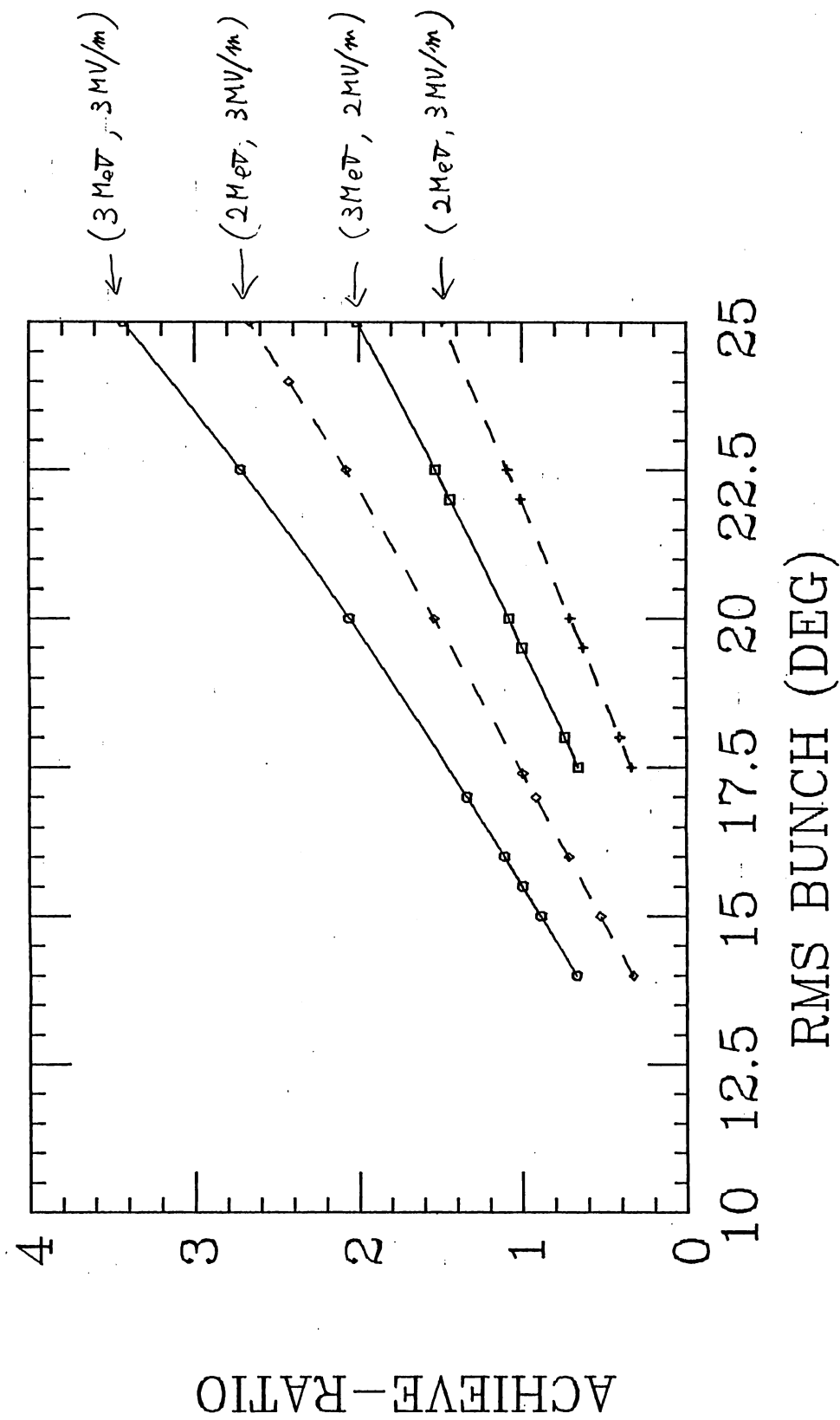


Fig. 1

$\sigma_p^t = 60$ , 201 MHz  
1750 keV injection

$E_{inj} = 0.4 \pi$  cm mrad

(1) 2 MV/m, 100 mA

(2) 1.5 MV/m, 100 mA

(3) 1.5 MV/m, 20 mA

# ACHIEVE-RATIO 201

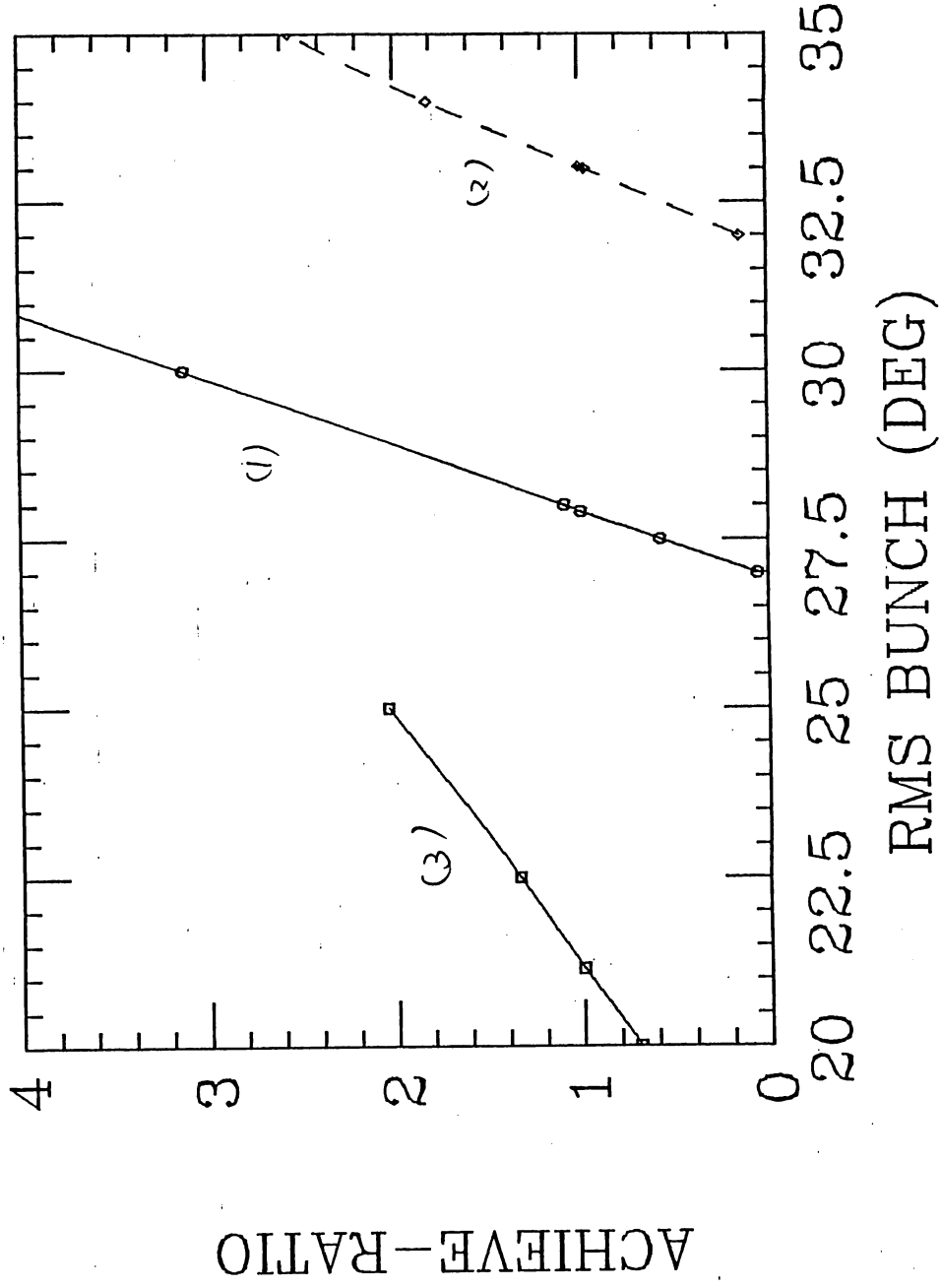


Fig. 2