大強度陽子リニアック

概要：equipartitioning を前提として、
縦と横の coupled envelope equations を
適用したJHP 大強度陽子リニアックの設計と、
その基礎となる新しい計算コードについて
解説する。

今後の質問先　kato@kekvax.kek.jp

予定

- LINAC - general
- Injection - effects of RFQ
- Emittance
- Envelope equation
- Beam distribution
- Equipartioning
- Acceleration process
- Collision
- JHP design

大強度陽子リニアック

- 高エネルギー　～　1 GeV
- 大強度　～　平均電流　1 mA

Beam Power

$10^{-3} \times 10^{9} = 1$ MW

PULSE LINAC　　Normal conducting Linac

$I_{ave} = I_p \times T \times f_r$　　$Q_0 \approx 10^4$　　**Pc = 10 MW**

JHP 50 mA x 200 μsec x 25 Hz = 250 μA

CW LINAC　　Superconducting Linac

$I_{ave} = I_p$　　$Q_0 \approx 10^9$　　**Pc = 100 W**

p.47

リニアックの簡単さ？

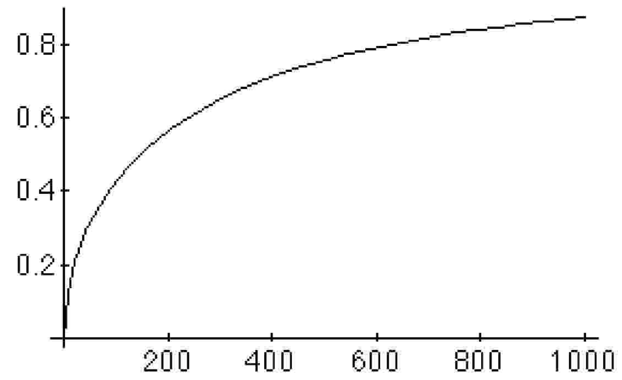
・ 直線加速器、単純な構成
・ どう作っても、そこそこには動くかも

BUT

なぜ＊＊＊＊では１０年以上たっても＊＊なビーム
しか供給できなかったか。

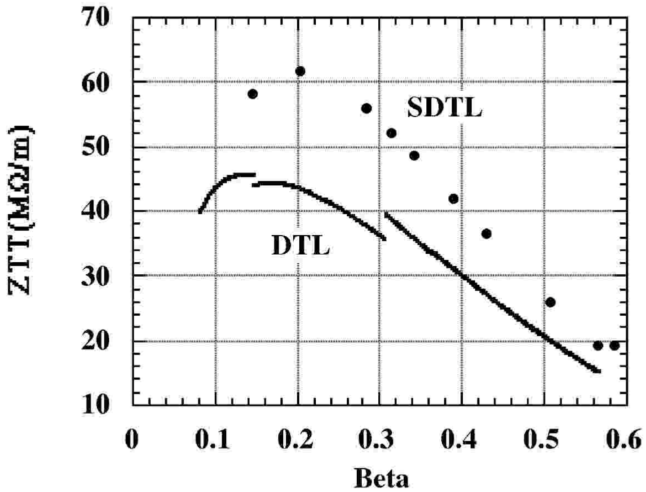
なぜ＊＊＊＊の高エネルギー
部エミッタンス増加は極端に
大きいのか。

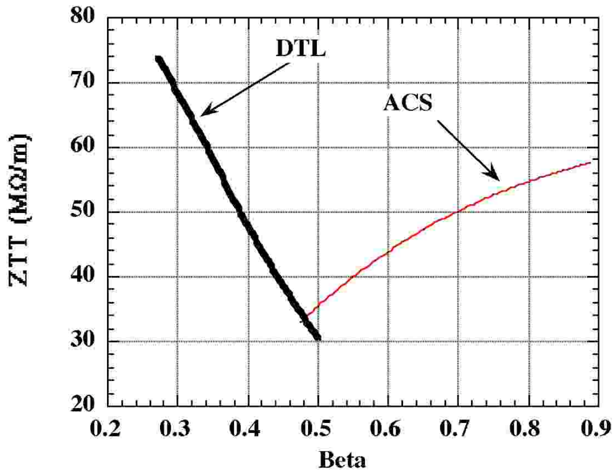
Proton Wp=938.28 MeV



ZTT

$$Z \propto \omega^{1/2} \quad KL \propto \omega^{1/2}$$

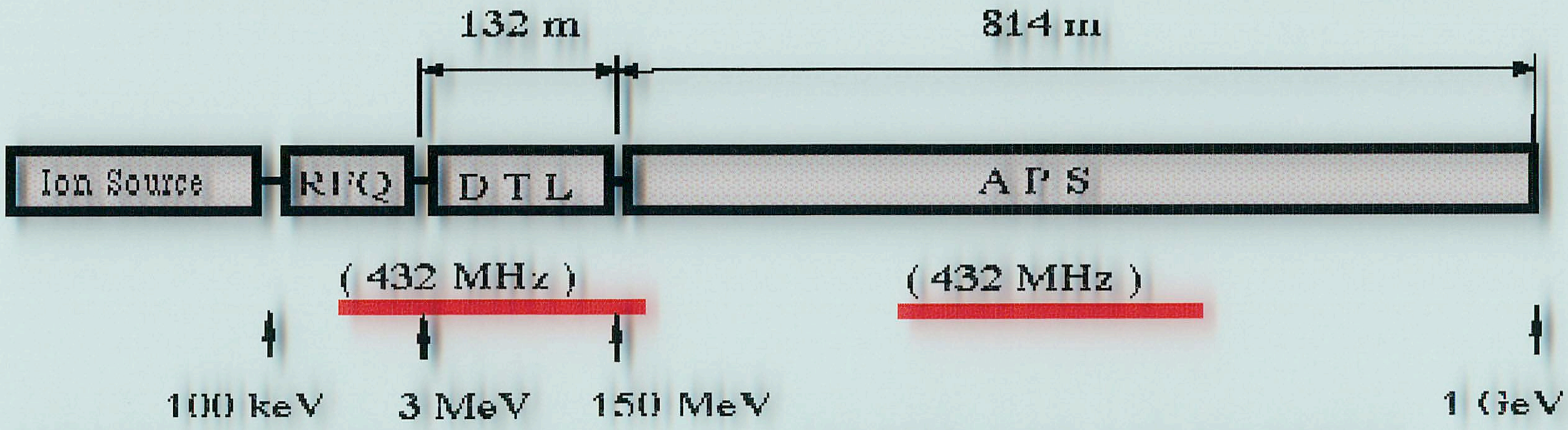




p.51, 52

p.49

A Proposal for CW-PLA 100 mA

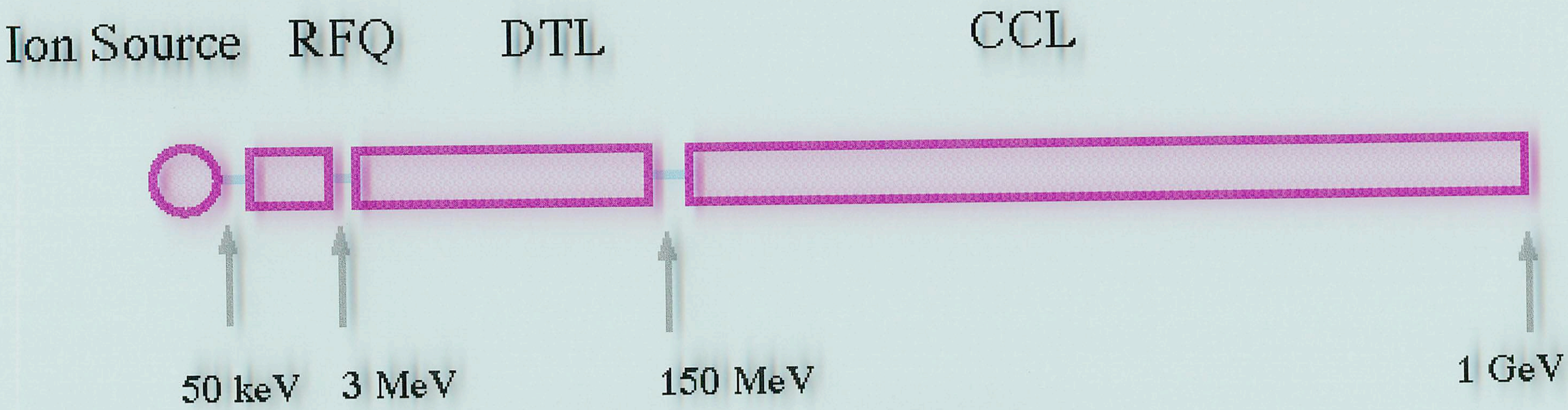


432 MHz DTL with 1.7 MV/m
*cell number 593
* Total cavity length 132.11 m
*Total rf power 3.89 MW
*Total rf power x 1.3 5.70 MW
*Beam power (100 mA) 14.5 MW

432 MHz CCL LINAC 910322
* APS with 50 mm bore radius
* E0 = 1.7 - 2.5 MV/m
* fs = -30 degrees
* Total cavity length 590.06 m
*Total cavity length with Q 814.01 m
*Total rf power 97.7 MW
*Total rf power x 1.2 117.2 MW
*Beam power (100 mA) 86.7 MW
*Total number of tanks 268
*Total number of cells 2328

p.52

1 GeV Proton Linac for JHP old design



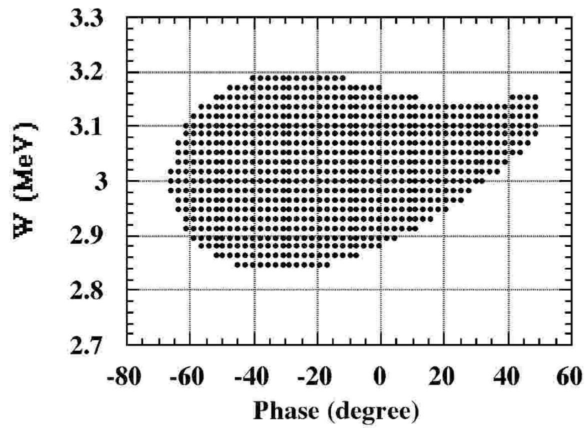
Ion Source	RFQ	DTL	CCL
20 mA	432 MHz	432 MHz	1296 MHz
1 πmm•mrad	2.7 m	79 m	411 m
	329 cells	13 tanks	152 tanks
	800 kW	342 cells	3576 cells
		9 MW	82 MW

p.52

Frequency transition

- LANL x4
- INR x5

I=qNf



$$\mu_\ell = \frac{q}{mc^2}\frac{1}{\beta_0^2\gamma_0^3}\frac{\rho_0 M_z}{\varepsilon_0}\frac{1}{k_\ell^2}$$

$$2\phi_s(1-\mu_\ell) \le \phi \le -\phi_s(1-\mu_\ell)$$

$$\Delta W_{max} = \pm\sqrt{-\frac{2}{3\pi}\lambda q E_m mc^2\beta_0^3\gamma_0^3\phi_s^3(1-\mu_\ell)^3}$$

p.12

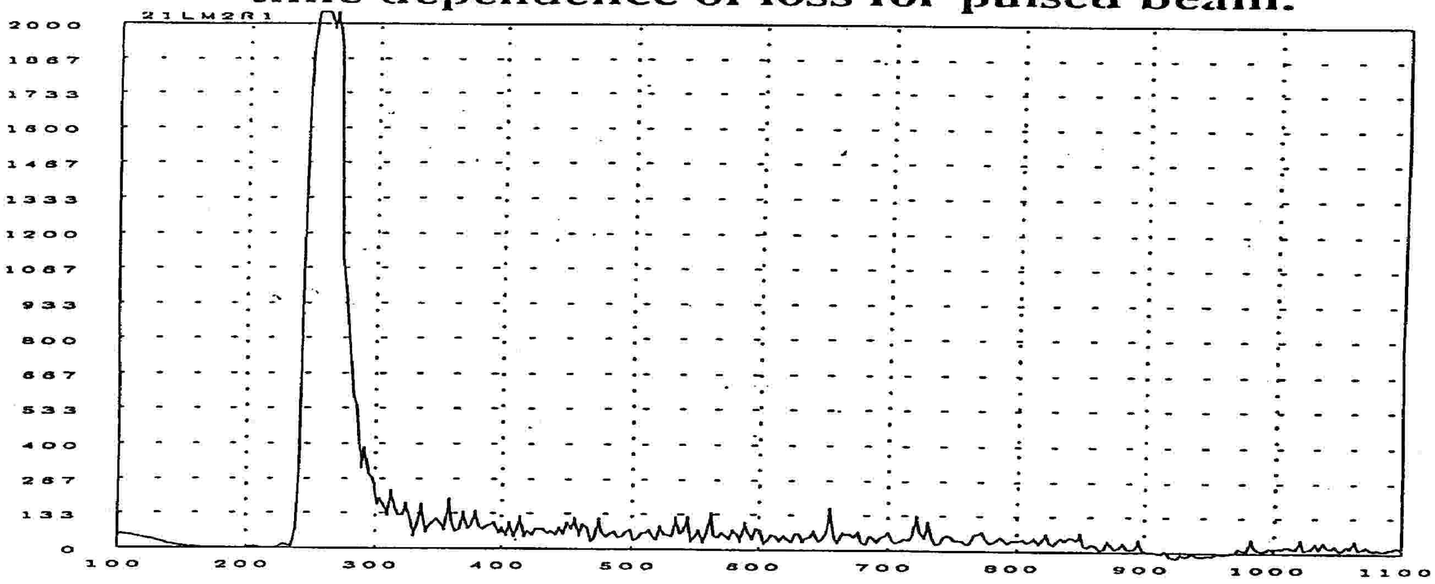
Beam loss problem

- マシンの運転がビーム損失に伴う放射化による運転制限とならないように。
- 定期的なメンテナンスが行える事
- ビーム損失は 10^{-4} - 10^{-5} に押さえる
- 高エネルギーにおいて連続して小さな損失が起こる可能性
  - ビームハロー
  - 種々のミスマッチ
  - 普通の計算では評価が難しい

PS Main ring では、夏の運転は７月８日で停止。これは夏作業の被爆線量を少なくする為。一方、ブースターは７月２６日まで運転した。

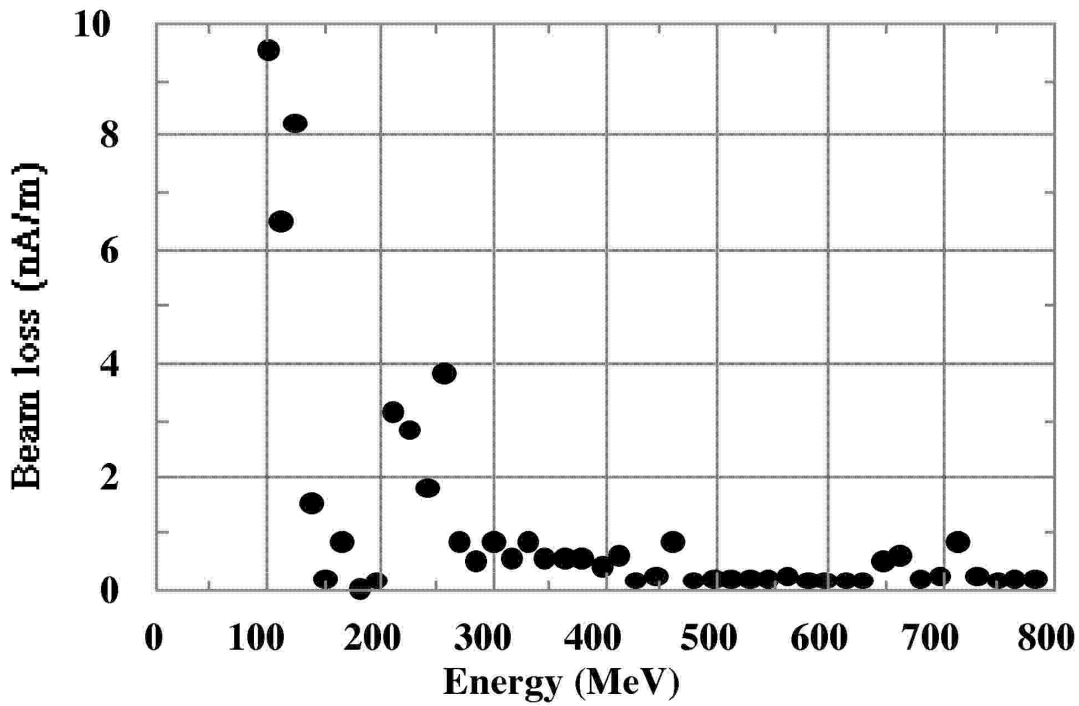
Transient Beam Loss

LAMPF beam-loss monitor readout, showing time dependence of loss for pulsed beam.



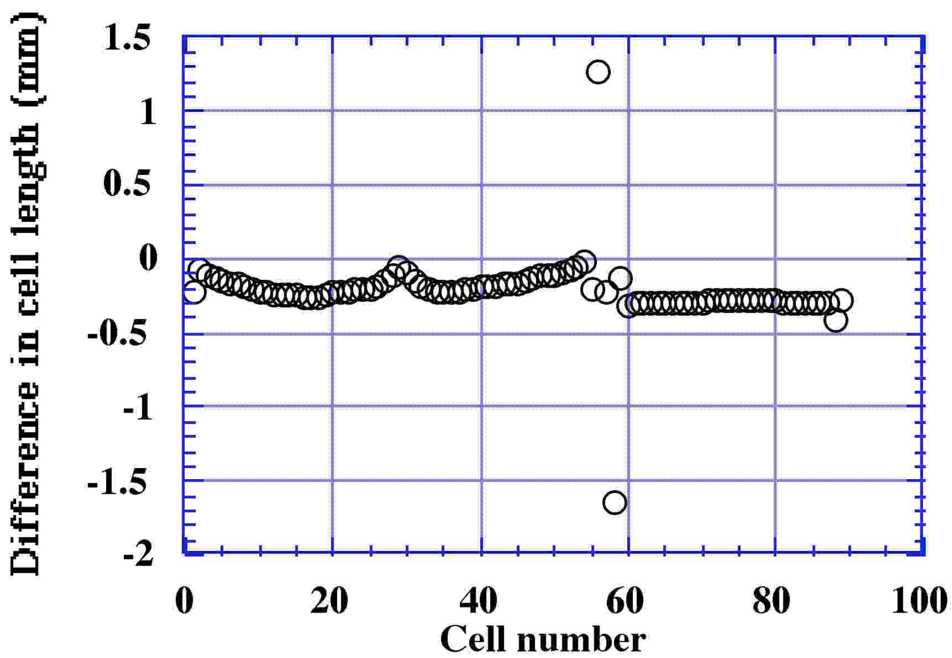
Los Alamos - Brookhaven

Beam Loss in High-Energy Structure

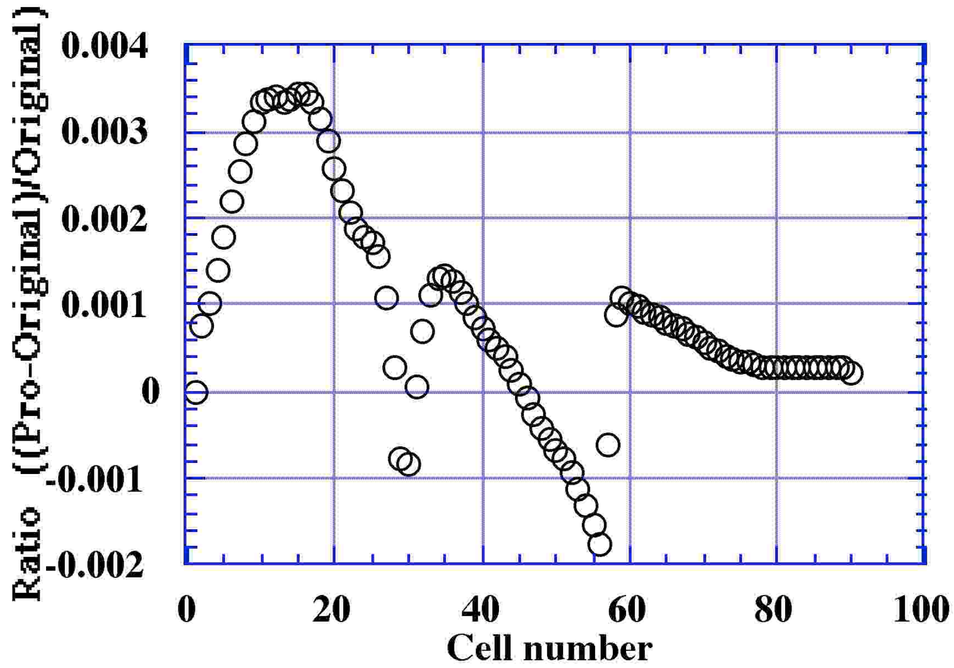


Beam loading

Comparison



Difference in cell length (mm)
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
20
40
60
80
100
Cell number



Ratio ((Pro-Original)/Original)
0.004
0.003
0.002
0.001
0
-0.001
-0.002
0
20
40
60
80
100
Cell number

基本式

for a unit cell

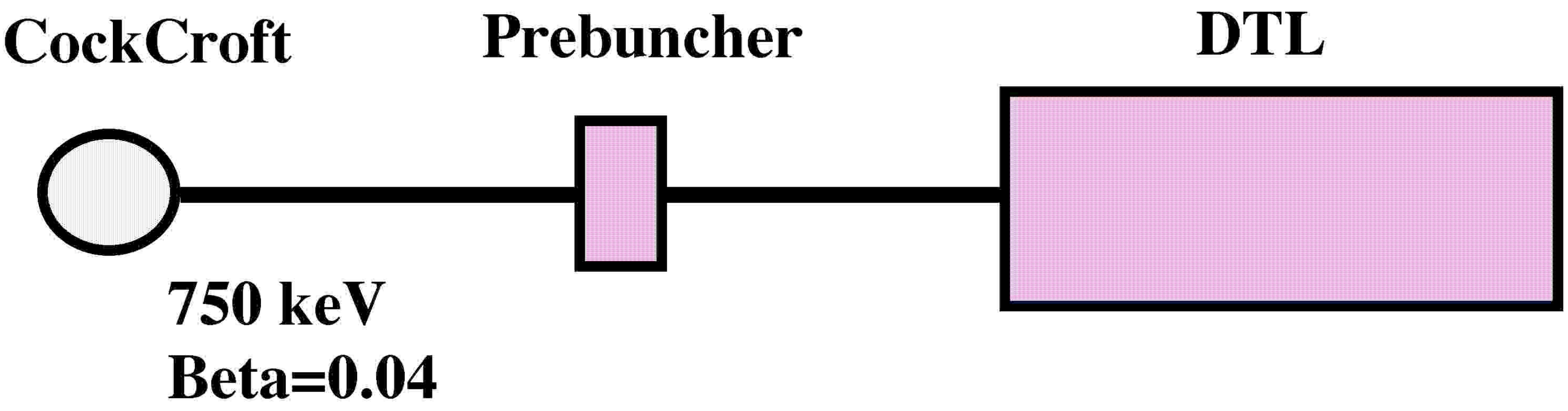
$$Q_0 = \frac{\omega U}{P_c} = \frac{2\pi \times 324 \times 10^6 \times 33.33 \times 10^{-3}(J)}{1171(W)} \approx 57900$$

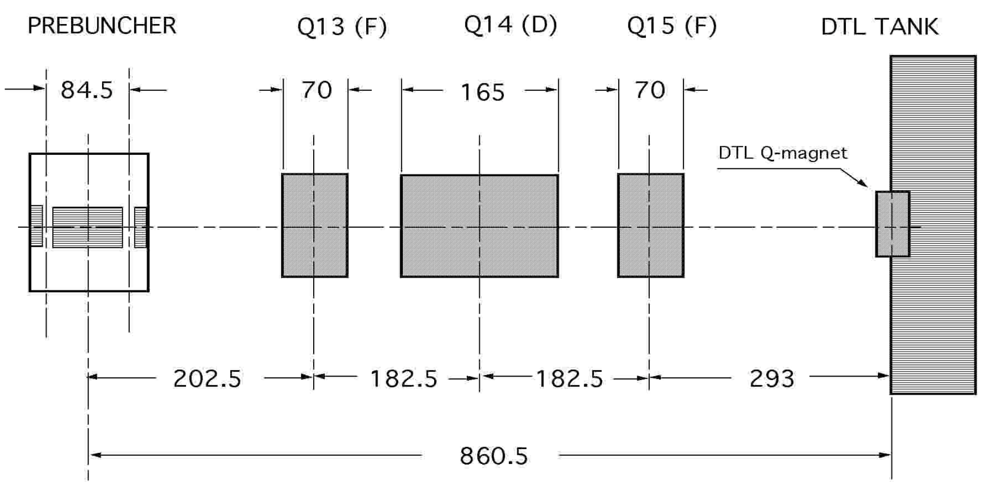
$$Z = \frac{E_0^2}{P_c / L} = \frac{\left(1 \times 10^6\right)^2 \times 0.0738}{1171} = 63\ M\Omega / m$$

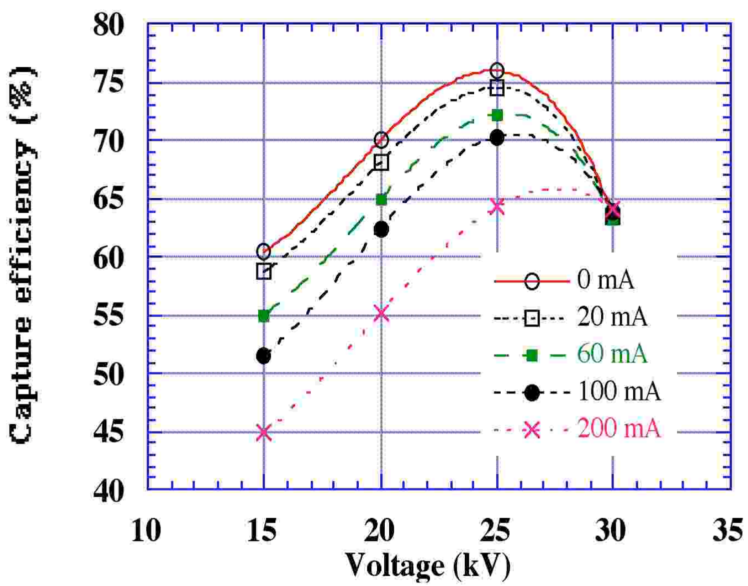
$$T = 0.728$$

$$ZTT = 33.4\ M\Omega / m$$

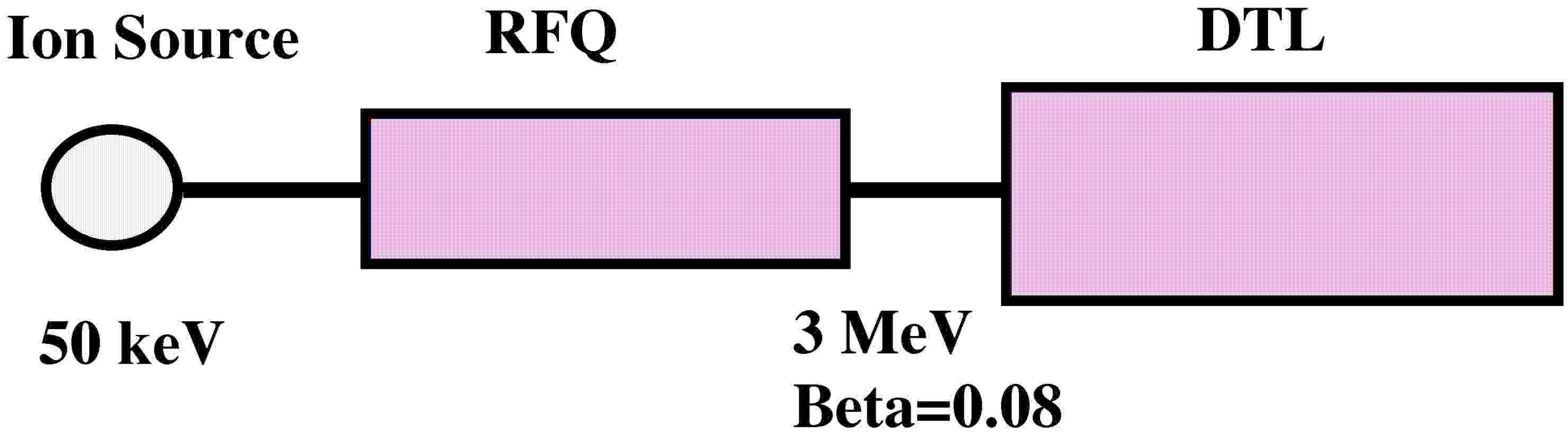
Old-type injection

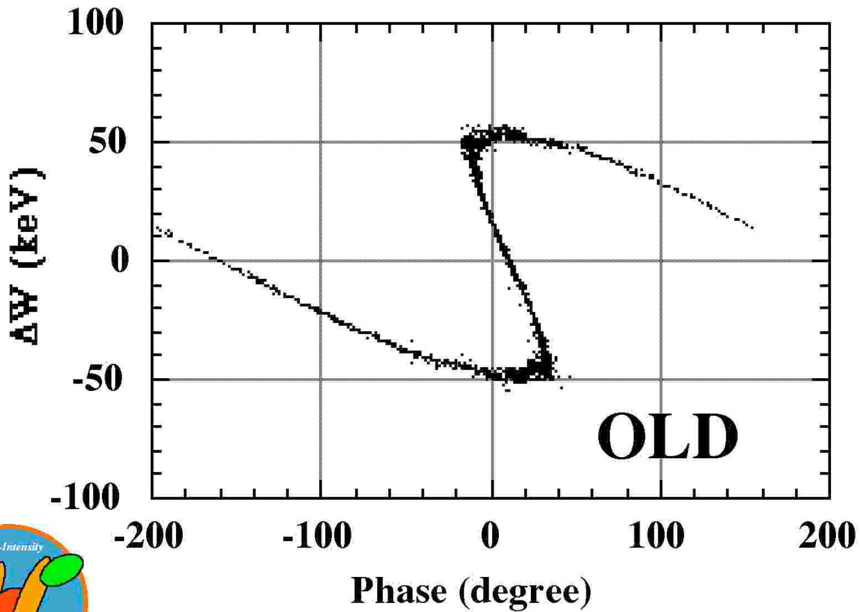


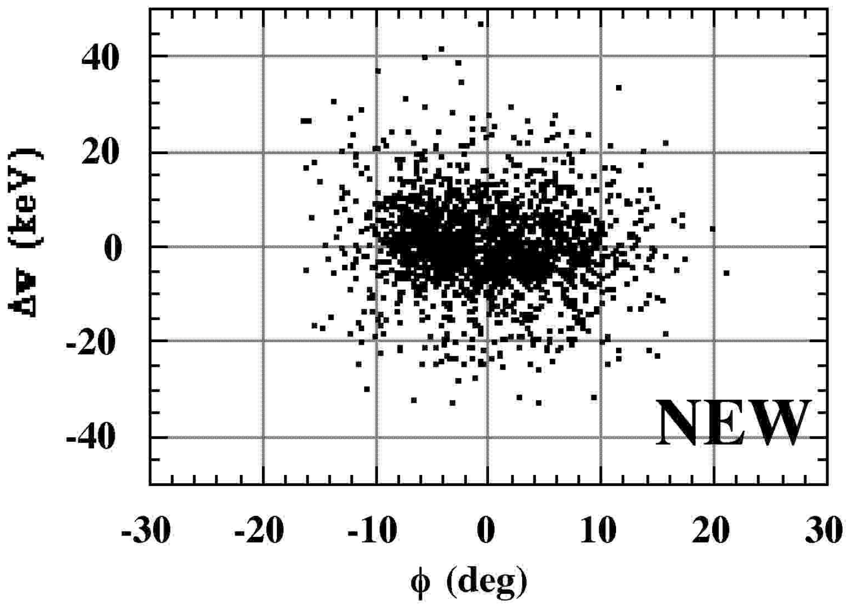




RFQ Impact


Ion Source
RFQ
DTL
50 keV
3 MeV
Beta=0.08


100
50
0
-50
-100
ΔW (keV)
-200
-100
0
100
200
Phase (degree)
OLD


40
20
0
-20
-40
Δw (keV)
-30
-20
-10
0
10
20
30
φ (deg)
NEW

Prebuncher with space charge

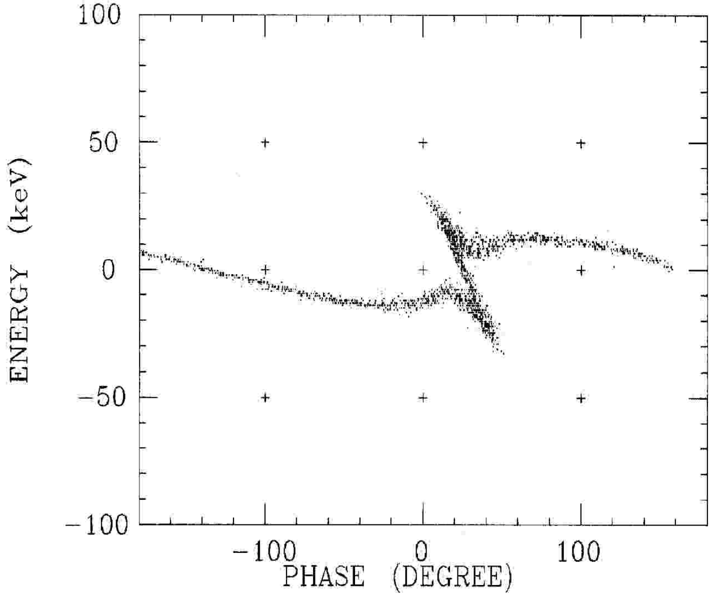
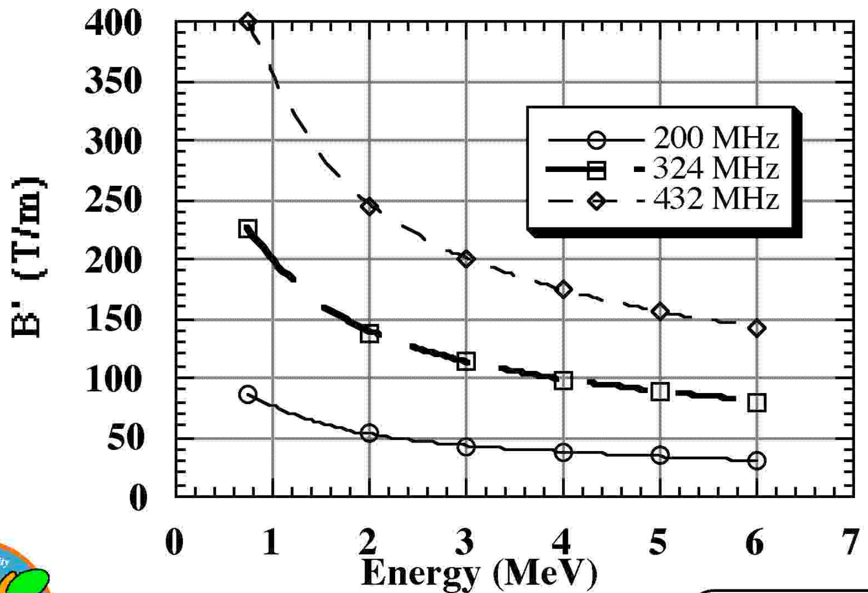


Fig. 17 Longitudinal emittance for a 100 mA beam at the DTL entrance. The buncher voltage is 25 kV.

RFQ Impact-2

$$B' \propto \frac{f^2}{\beta}$$

- Higher DTL injection energy
- Higher capture efficiency
- better beam quality in longitudinal motion
- Comparison between experimental data and theoretical prediction



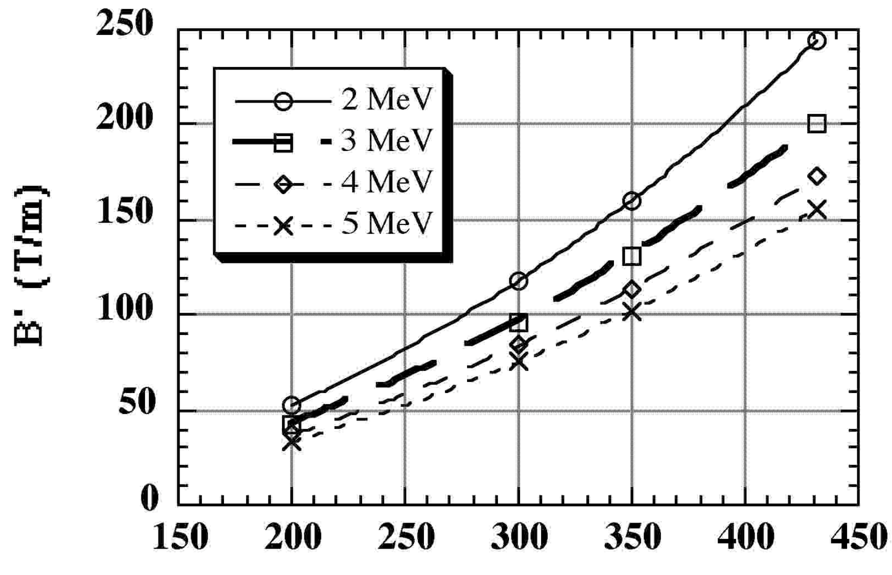
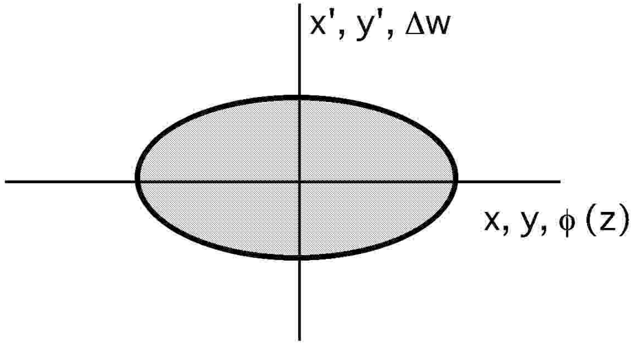


図６－２

p.42, 43

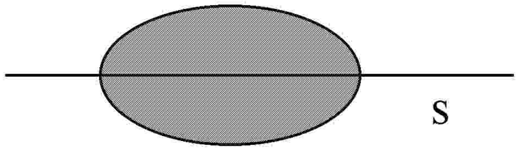
Emittance-1



$$A_x = \frac{1}{p}\iint dx\,dp_x = \frac{1}{\beta\gamma mc}\iint dx\,dp_x$$

$$x' = \frac{dx}{ds} = \frac{v_x}{v_0} \approx \frac{p_x}{p}$$

BEAM



$$\varepsilon_x = \frac{A_x}{\pi} = x_{max}x'_{max}$$

$$\varepsilon_{xn} = \beta\gamma\varepsilon_x$$

p.3, 4

rms emittance

$$\tilde{\varepsilon}_x = 4\sqrt{\overline{x^2}\;\overline{x'^2} - \overline{xx'}^2}$$

$$\tilde{\varepsilon}_x = \sqrt{\overline{x^2}\;\overline{x'^2}} = \frac{x_{max}x'_{max}}{4}$$

$$\tilde{\varepsilon}_x = \tilde{x}\tilde{x}' = \tilde{x}\frac{\tilde{v}_x}{v_0}$$

$$\overline{x^2} = \frac{1}{N}\int x^2 f(x,p,t)\, dxdp$$

$$\tilde{\varepsilon}_{xn} = \tilde{x}\left(\frac{k_B T_\perp \gamma_0}{mc^2}\right)^{1/2}$$

$$\tilde{v}_x = \tilde{v}_y = \tilde{v}_z = \left(\frac{k_B T}{m}\right)^{1/2}$$

p.4

Longitudinal emittance

$$\tilde{\varepsilon}_z = \tilde{z}\frac{\tilde{p}_z}{p_0} \qquad \tilde{\varepsilon}_{nz} = \beta_0\gamma_0\tilde{\varepsilon}_z = \tilde{z}\frac{\tilde{p}_z}{mc}$$

$$z = s - s_0$$

$$\Delta v_z = v - v_0$$

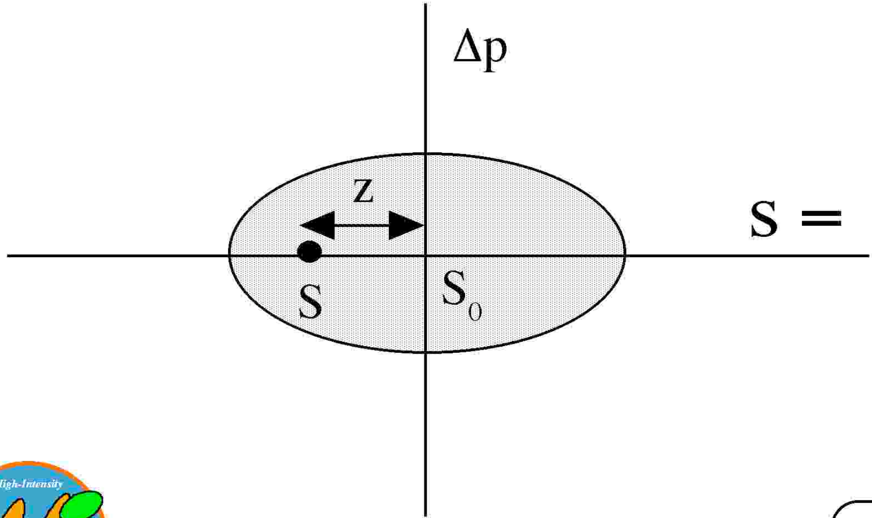
$$\Delta p_z = p - p_0$$

$$p_0 = mc\beta_0\gamma_0$$

$$z' = \frac{dz}{ds} = \frac{\Delta v_z}{v_0} = \frac{1}{\gamma_0^2}\frac{\Delta p_z}{p_0}$$

$$\tilde{\varepsilon}_{zz'} = \tilde{z}\tilde{z}' = \frac{1}{\gamma_0^2}\tilde{\varepsilon}_z$$

$$\tilde{\varepsilon}_{nz} = \beta_0\gamma_0^3\tilde{\varepsilon}_{zz'}$$



$$s = vt$$

$$\varepsilon_{\Delta\phi\Delta W} = \Delta\phi\cdot\Delta W = \frac{2\pi}{\lambda}mc^2\beta_0\gamma_0\Delta z\frac{\Delta p}{p_0} = \frac{2\pi W_p}{\lambda}\varepsilon_{nz}$$

p.13, 14

Beam distribution

uniform
Gaussian
Waterbag
K-V
Hollow

Maxwell-Boltzmann distribution

$$f(H) = f_0 \exp\left(-\frac{H}{k_B T}\right) \qquad H = \frac{m}{2}\left(v_x^2 + v_y^2 + v_z^2\right) + q\phi$$

$$f(r, v) = f_0 \exp\left[-\frac{m\left(v_x^2 + v_y^2 + v_z^2\right)}{2k_B T} - \frac{q\phi(x, y, z)}{2k_B T}\right]$$

$$f(H_\perp, H_\parallel) = A \exp\left(-\frac{H_\perp}{k_B T_\perp}\right) \exp\left(-\frac{H_\parallel}{k_B T_\parallel}\right)$$

p.7~

Background-1

Liouville's theorem

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \sum_{i=1}^{3}\left(\frac{\partial f}{\partial q_i}\dot{q}_i + \frac{\partial f}{\partial p_i}\dot{p}_i\right) = 0$$

$$\iint d^3q\, d^3p = \text{const}$$

Collisionless Vlasov equation

$$\frac{\partial f}{\partial t} + \sum_{i=1}^{3}\left(\frac{\partial f}{\partial q_i}\dot{q}_i + (qE + qv \times B)_i \frac{\partial f}{\partial P_i}\right) = 0$$

$$\mathbf{F}_i = \frac{q^2}{4\pi\varepsilon_0}\sum_{j\neq i}^{N}\frac{\mathbf{r}_{ij}}{\mathbf{r}_{ij}^3}$$

Collision term

$$\mathbf{F}_i = -\frac{\partial}{\partial \mathbf{r}} q\phi_s + \underline{\frac{q^2}{4\pi\varepsilon_0}\sum_{j}\frac{r_{ij}}{r_{ij}^3}}$$

$$\frac{\partial f}{\partial t} + \sum_{i=1}^{3}\left(\frac{\partial f}{\partial q_i}\dot{q}_i - \frac{q}{m}(E_a + E_s + v \times B)_i \frac{\partial f}{\partial \dot{q}_i}\right)$$

$$= 0$$

$$= \left[\frac{\partial f}{\partial t}\right]_c \neq 0$$ **Boltzmann equation**

p.7~

Background-2

衝突項を含む **Fokker-Planck equation**

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + v \cdot \frac{\partial f}{\partial r} + \frac{q}{m}\frac{\partial \phi}{\partial r} \cdot \frac{\partial f}{\partial v} = \beta_f \frac{\partial (\mathbf{v} f)}{\partial \mathbf{v}} + D \frac{\partial^2 f}{\partial v^2}$$

＝０の定常解は **Gauss** 分布であり、**Maxwell** の速度分布と一致する

$$f(\mathbf{v}) = \left(\frac{m}{2\pi k_B T} \right)^{3/2} \exp\left(-\frac{m\left(v_x^2 + v_y^2 + v_z^2\right)}{2k_B T} \right)$$

$$\frac{D}{\beta_f} = \frac{k_B T}{m}$$

p.7~

Temperature

$$f(H_\perp, H_\parallel) = A\exp\left(-\frac{H_\perp}{k_B T_\perp}\right)\exp\left(-\frac{H_\parallel}{k_B T_\parallel}\right)$$

$$k_B T_\perp = \gamma_0 m \Delta\tilde{v}_x^2$$

$$k_B T_\parallel = \gamma_0^3 m \Delta\tilde{v}_z^2$$

$$\tilde{\varepsilon}_{nx} = \tilde{x}\left(\frac{\gamma_0 k_B T_\perp}{mc^2}\right)^{1/2}$$

$$\tilde{\varepsilon}_{nz} = \tilde{z}\left(\frac{\gamma_0^3 k_B T_\parallel}{mc^2}\right)^{1/2}$$

laboratory frame vs beam frame

$$T_\ell = \frac{T_b}{\gamma_0}$$

p.14

Space-charge force 2D beam

Uniform distribution

$$I = Nqv$$

$$F_r = qE_r - qv\mu_0 H_\phi = qE_r\left(1-\beta^2\right)$$

$r < a$

$$E_r = \frac{Nq}{2\pi\varepsilon_0 a^2} r$$

$$H_\phi = \frac{Nqv}{2\pi a^2} r$$

Field energy

$$W = \frac{1}{2}\int_0^b \left(\varepsilon_0 E^2 - \mu_0 H^2\right) 2\pi r dr$$

$r > a$

$$E_r = \frac{Nq}{2\pi\varepsilon_0}\frac{1}{r}$$

$$\mathbf{W_u} = W_0\left(1 + 4\ell n\frac{b}{a}\right)$$

$$W_0 = \frac{(Nq)^2}{16\pi\varepsilon_0}\left(1-\beta^2\right) = \frac{(Nq)^2}{16\pi\varepsilon_0\gamma^2}$$

$$H_\phi = \frac{Nqv}{2\pi}\frac{1}{r}$$

Non-linear field energy $U = W - W_u$

p.9

Space-charge force 3D bunch

Uniform distribution

$$I = \frac{4\pi}{3} a^2 z_m \rho_0 f = Qf = qNf$$

$$E_{\rho x} = \frac{\rho_0}{\varepsilon_0}\left(1-\beta_0^2\right) M_x x$$

$$E_{\rho y} = \frac{\rho_0}{\varepsilon_0}\left(1-\beta_0^2\right) M_y y$$

$$E_{\rho z} = \frac{\rho_0}{\varepsilon_0} M_z z$$

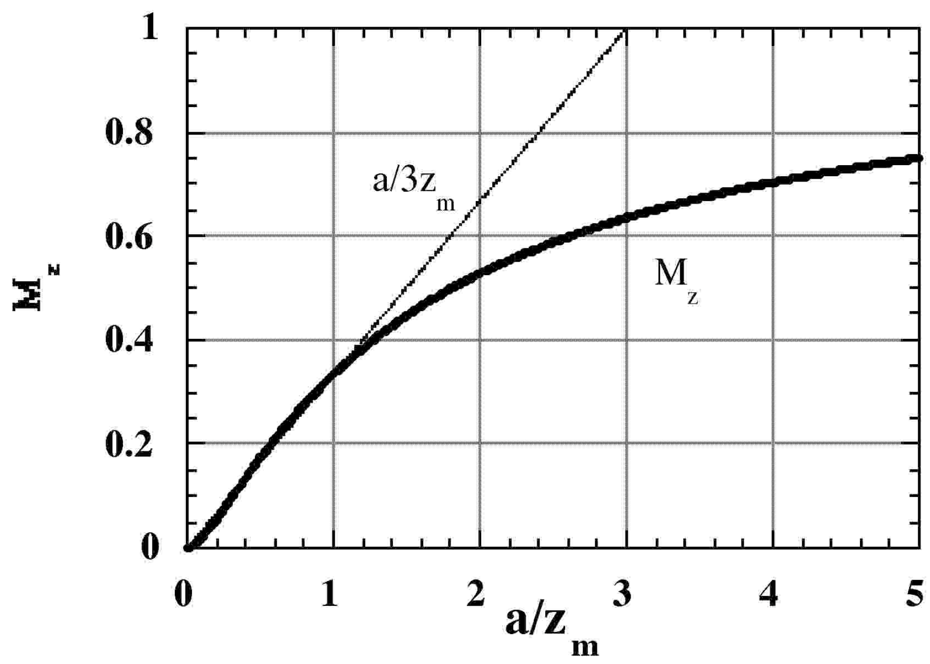


図2 - 2 Ellipsoid form factor M_z.

p.10

Longitudinal motion with space charge

$$\frac{1}{\beta_0^3\gamma_0^3}\frac{d}{ds}\left(\beta_0^3\gamma_0^3\frac{d\Delta\phi}{ds}\right) = -k_\ell^2\left(1+\frac{\Delta\phi}{2\phi_s}-\mu_\ell\right)\Delta\phi$$

$$\frac{1}{\beta_0^3\gamma_0^3}\frac{d}{dz}\left(\beta_0^3\gamma_0^3\frac{d\Delta\phi}{dz}\right) = -k_{\ell\rho}^2\left(1+\frac{\Delta\phi}{2\phi_{s\rho}}\right)\Delta\phi$$

$$k_\ell^2 = -\frac{q}{mc^2}\frac{2\pi}{\lambda}\frac{E_m\sin\phi_s}{\beta_0^3\gamma_0^3}$$

$$k_{\ell\rho}^2 = k_\ell^2\left(1-\mu_\ell\right)$$

$$2\phi_s\left(1-\mu_\ell\right) \le \phi \le -\phi_s\left(1-\mu_\ell\right)$$

$$\mu_\ell = \frac{q}{mc^2}\frac{1}{\beta_0^2\gamma_0^3}\frac{\rho_0 M_z}{\varepsilon_0}\frac{1}{k_\ell^2}$$

$$\Delta W_{max} = \pm\sqrt{-\frac{2}{3\pi}\lambda q E_m mc^2\beta_0^3\gamma_0^3\phi_s^3\left(1-\mu_\ell\right)^3}$$

Transverse motion with space charge

$$\frac{d}{dz}\left(\beta_0\gamma_0\frac{dx}{dz}\right) = \frac{q}{mc^2}\left(\frac{-\pi}{\lambda}\frac{E_m\sin\phi}{\beta_0^2\gamma_0^2} + \frac{\rho_0}{\varepsilon_0}\frac{M_x}{\beta_0\gamma_0^2} - cB'\right)x$$

$$\frac{1}{\beta_0\gamma_0}\frac{d}{ds}\left(\beta_0\gamma_0\frac{dx}{ds}\right) = -k_t^2(1-\mu_t)x = k_{t\rho}^2 x$$

$$k_{t\rho}^2 = k_t^2(1-\mu_t)$$

$$\mu_t = \frac{q}{mc^2}\frac{\rho_0}{\varepsilon_0}\frac{M_x}{\beta_0^2\gamma_0^3}\frac{1}{k_t^2}$$

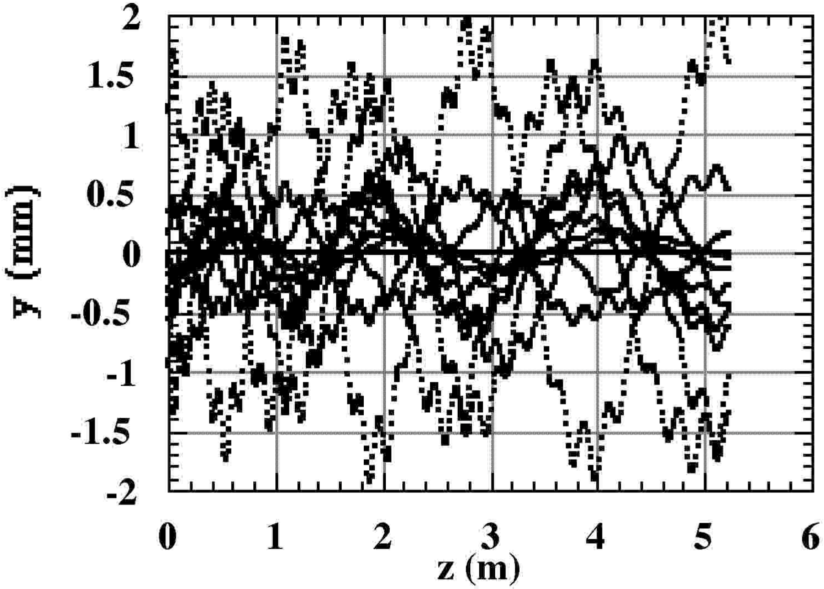
$$k_t^2 = k_{t,q}^2 - k_{t,rf}^2$$

$$k_{t,q}^2 = \frac{q}{mc^2}\frac{cB'}{\beta_0\gamma_0}$$

$$k_{t,rf}^2 = -\frac{q}{mc^2}\frac{\pi}{\lambda}\frac{E_m\sin\phi}{\beta_0^3\gamma_0^3}$$

p.12

Beam envelope



$$z'' + k_{z0}^2 z - \frac{K_L}{z_m^3} z = 0$$

$$z''_m + k_{z0}^2 z_m - \frac{K_L}{z_m^2} - \frac{\varepsilon_{zz'}^2}{z_m^3} = 0$$

$$a'' + k_{x0}^2 a - \frac{3}{2} \frac{N r_c}{\beta_0^2 \gamma_0^3} \frac{1}{a z_m} \left(1 - M_z\right) - \frac{\varepsilon_x^2}{a^3} = 0$$

$$X'' + k_{x0}^2 X - \frac{\tilde{\varepsilon}_x^2}{X^3} - \frac{\overline{x F_x}}{X} = 0$$

p.5, 15

Envelope equation and rms matched beam

$$X'' + k_{x0}^2 X - \frac{\tilde{\varepsilon}_x^2}{X^3} - \frac{K}{4X} = 0$$

$$a = \sqrt{2}r = 2X = 2Y \qquad \varepsilon = 4\tilde{\varepsilon}_x$$

rms matched beam

$$a'' + k_0^2 a - \frac{\varepsilon^2}{a^3} - \frac{K}{a} = 0$$

$$a = \text{const}$$

$$k_0^2 a - \frac{\varepsilon^2}{a^3} - \frac{K}{a} = 0$$

$$ka^2 = \varepsilon$$

$$k^2 = \left(\frac{2\pi}{\lambda_f}\right)^2 = k_0^2 - \frac{K}{a^2}$$

$$\frac{k}{k_0} = \frac{1}{\sqrt{2}} = 0.707$$

$$\mu = 1 - \frac{k^2}{k_0^2} = 0.5$$

$Ka^2 > \varepsilon^2$ **space-charge dominated**

$Ka^2 < \varepsilon^2$ **emittance dominated**

p.17

Beam distribution-1

$$f = f_0 \exp\left(-\frac{H_\perp}{k_B T_\perp}\right)$$

$$H_\perp = \frac{m}{2}\left(v_x^2 + v_y^2\right) + \phi_a + \phi_s$$

$$\phi_a = \frac{1}{2}\gamma_0 m v_0^2 k_0^2 r^2$$

$$f = f_0 \exp\left[-\frac{\gamma_0 m v_0^2\left(r'^2_\perp + k_0^2 r^2\right)}{2k_B T_\perp} - \frac{q\phi_s(r)}{k_B T_\perp \gamma_0^2}\right]$$

$$n(r) = n(0)\exp\left[-\frac{\gamma_0 m v_0^2 k_0^2 r^2}{2k_B T_\perp} - \frac{q\phi_s(r)}{k_B T_\perp \gamma_0^2}\right]$$

$$E_s(r) = \frac{q}{\varepsilon_0 r}\int_0^r r\, n(r)\, dr$$

$$\phi_s(r) = -\int_0^r E_s(r)\, dr$$

p.18

Beam distribution-2

$$n(r) = n(0)\exp\left[-\frac{\gamma_0 m v_0^2 k_0^2 r^2}{2k_B T_\perp} - \frac{q\phi_s(r)}{k_B T_\perp \gamma_0^2}\right]$$

Space-charge limit

$$\frac{\gamma_0 m v_0^2 k_0^2 r^2}{2} = -\frac{q\phi_s(r)}{\gamma_0^2} = \frac{q^2 n_0 r^2}{4\varepsilon_0 \gamma_0^2}$$

$$n(r) = n_0 \quad \text{for } 0 \le r \le a_0$$

$$n(r) = 0 \quad \text{for } r > a_0$$

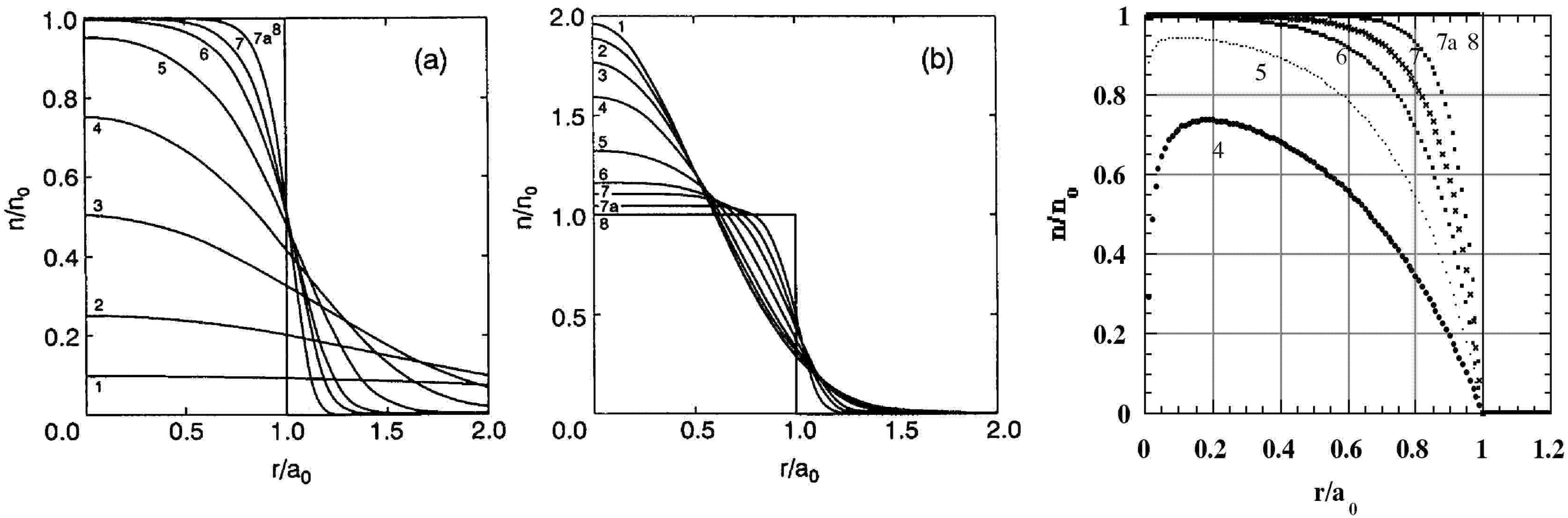
Zero current

$$\phi_s = 0$$

$$n(r) = n(0)\exp\left[-\frac{\gamma_0 m v_0^2 k_0^2}{2k_B T_\perp} r^2\right]$$

p.19

Beam distribution-3



p.20

Beam distribution-Table

表3 - 1 Relevant parameters for the radial Boltzmann density profiles of Figure 3 - 2.

Curve	$n(0)/n_0$	$\lambda_D(0)/a_0$	$\tilde{r}/\tilde{r}_0$	$\bar{\lambda}_D/a$	Ka^2/ε^2	k/k_0	$n(0)/n_0$ for $\tilde{r}=r_0$
1	0.1	4.82	4.43	1.52	0.054	0.974	1.96
2	0.25	1.81	2.75	0.905	0.153	0.931	1.89
3	0.5	0.795	1.88	0.562	0.396	0.846	1.77
4	0.75	0.432	1.46	0.374	0.893	0.727	1.60
5	0.95	0.229	1.18	0.223	2.51	0.534	1.32
6	0.995	0.145	1.08	0.144	6.00	0.378	1.16
7	0.9995	0.107	1.04	0.107	10.9	0.290	1.08
7a	0.999995	0.0710	1.02	0.0710	24.8	0.197	1.04
8	1	0	1	0	∞	0	1

p.20

Effects of acceleration on beam distribution

$$\frac{Ka^2}{\varepsilon^2} = \frac{k_0^2}{k^2} - 1$$

$$\frac{Ka^2}{\varepsilon^2} = \frac{I}{I_0}\frac{2a^2}{\beta\gamma\varepsilon_n^2} = \frac{I}{I_0}\frac{\tilde{x}^2}{2\beta\gamma\tilde{\varepsilon}_n^2}$$

$$I_0 = \frac{4\pi\varepsilon_0 mc^3}{q} \approx \frac{1}{30}\frac{mc^2}{q} \approx 31\ \text{MA}$$

2 MeV $\beta\gamma = 0.065$ $\frac{Ka^2}{\varepsilon^2} = 24.8$

$\tilde{\varepsilon}_n = \beta\gamma\tilde{\varepsilon} = 0.2$ mm – mrad $\frac{k}{k_0} = 0.2$

$\tilde{x} = 2$ mm $B_f = \frac{\bar{I}}{\hat{I}} = 0.1$

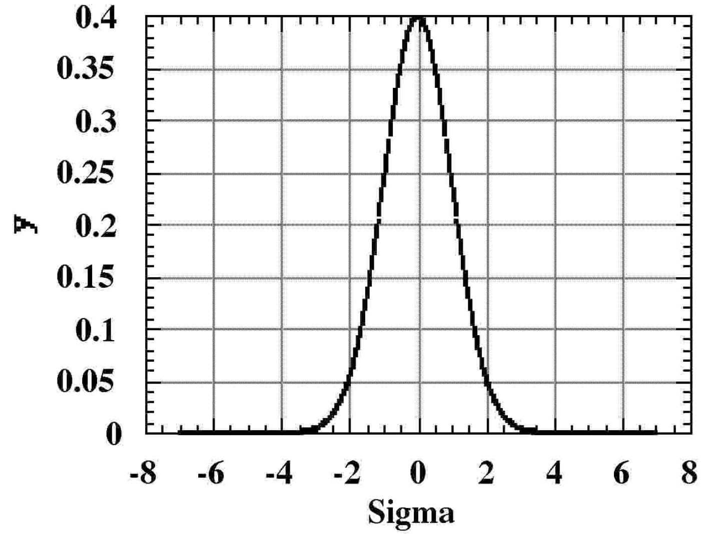
$I = 100$ mA $I_{bp} = 1$ A $\bar{\lambda}_D / a = 0.07$

1000 MeV $\frac{Ka^2}{\varepsilon^2} = 0.89$

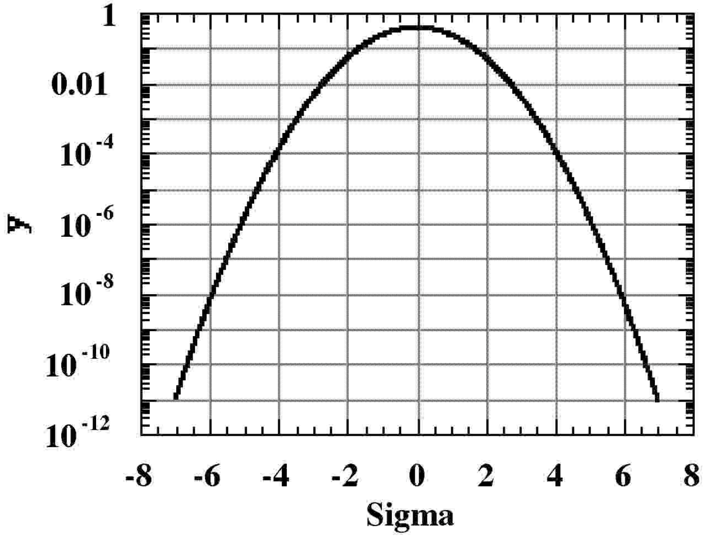
$\beta\gamma = 1.81$ $\frac{k}{k_0} = 0.73$ $\bar{\lambda}_D / a = 0.37$

p.19

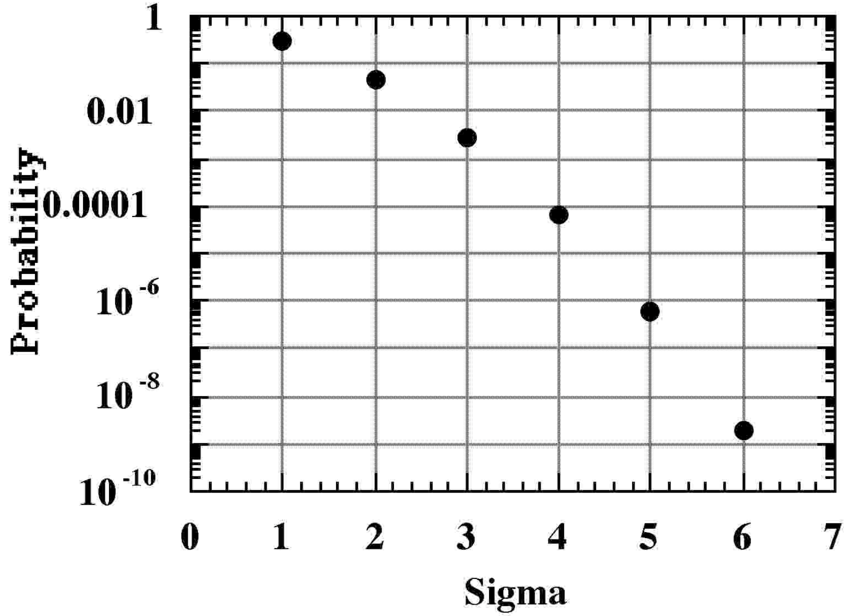
Gauss のすそ



0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
y
-8
-6
-4
-2
0
2
4
6
8
Sigma



1
0.01
10 -4
10 -6
10 -8
10 -10
10 -12
y
-8
-6
-4
-2
0
2
4
6
8
Sigma



1
0.01
0.0001
10 -6
10 -8
10 -10
Probability
0
1
2
3
4
5
6
7
Sigma

Equipartitioning

$$\tilde{\varepsilon}_{nx} = \tilde{x}\left(\frac{\gamma_0 k_B T_\perp}{mc^2}\right)^{1/2}$$

$$\frac{T_\perp}{T_{||}} = \gamma_0^2 \frac{\tilde{\varepsilon}_{nx}^2}{\tilde{\varepsilon}_{nz}^2}\frac{\tilde{z}^2}{\tilde{x}^2} = \gamma_0^2 \frac{\tilde{\varepsilon}_{nx}^2}{\tilde{\varepsilon}_{nz}^2}\frac{z_m^2}{a^2}$$

$$\tilde{\varepsilon}_{nz} = \tilde{z}\left(\frac{\gamma_0^3 k_B T_{||}}{mc^2}\right)^{1/2}$$

$$T_\perp = T_{||}$$

$$\gamma_0 \frac{\varepsilon_{nx}}{\varepsilon_{nz}}\frac{z_m}{a} = 1 \qquad \frac{k_x \varepsilon_{nx}}{k_z \varepsilon_{nz}} = 1 \qquad \varepsilon_{nx}\sigma_x = \varepsilon_{nz}\sigma_z$$

p.37

Emittance equation-1

$$\frac{d^2\mathbf{r}}{ds^2} = \frac{1}{m\gamma v^2}\mathbf{F} = \frac{-k}{m\gamma v^2}\mathbf{r} + \frac{q\mathbf{E}}{m\gamma v^2} = -K\mathbf{r} + \frac{q\mathbf{E}_e}{m\gamma^3 v^2}$$

$$\nabla \bullet \mathbf{E}_e = \frac{q}{\varepsilon_0} n(x, y, z, s)$$

$$\varepsilon_x = 4\sqrt{\overline{x^2}\;\overline{x'^2} - \overline{xx'}^2}$$

$$\frac{1}{\overline{x^2}}\frac{d\varepsilon_x^2}{ds} + \frac{1}{\overline{y^2}}\frac{d\varepsilon_y^2}{ds} + \frac{1}{\overline{z^2}}\frac{d\varepsilon_z^2}{ds}$$

$$I_u = -\frac{1}{Nq}\frac{dW_u}{ds}$$

$$= \frac{32q}{m\gamma^3 v^2}\left[-\frac{1}{Nq}\frac{dW}{ds} - \frac{\varepsilon_0}{Nq}\iint_S \phi \frac{\partial}{\partial s} E_n da - \frac{1}{2}\left(\frac{1}{\overline{x^2}}\frac{d\overline{x^2}}{ds}\overline{xE_x} + \frac{1}{\overline{y^2}}\frac{d\overline{y^2}}{ds}\overline{yE_y} + \frac{1}{\overline{z^2}}\frac{d\overline{z^2}}{ds}\overline{zE_z} \right) \right]$$

p.25~

Emittance equation-2

$$\frac{1}{\overline{\overline{x^2}}}\frac{d\varepsilon_x^2}{ds}+\frac{1}{\overline{\overline{y^2}}}\frac{d\varepsilon_y^2}{ds}+\frac{1}{\overline{\overline{z^2}}}\frac{d\varepsilon_z^2}{ds}=-\frac{32}{m\gamma^3 v^2 N}\left(\frac{dW}{ds}-\lambda_3\frac{dW_u}{ds}\right)$$

- source = difference in field energy
- charge redistribution
- emittance transfer - equipartitioning
- W for potential energy or something else
- with envelope equations

p.29~

Emittance equation-3

$$\frac{d}{ds}\left(\overline{x'^2}+\overline{y'^2}+\overline{z'^2}\right)+\left[k_x(s)\frac{d\overline{x^2}}{ds}+k_y(s)\frac{d\overline{y^2}}{ds}+k_z(s)\frac{d\overline{z^2}}{ds}\right]+\frac{2}{m\gamma^3v^2N}\left(\frac{dW}{ds}+\varepsilon_0\iint_S\phi\,\frac{\partial}{\partial s}E_n\,da\right)=0$$

$$T=\frac{m\gamma v^2}{2}\left(\overline{x'^2}+\overline{y'^2}+\overline{z'^2}\right)$$

$$V_{ex}=\frac{m\gamma v^2}{2}\left(k_x\overline{x^2}+k_y\overline{y^2}+k_z\overline{z^2}\right)$$

$$T+V_{ex}+\frac{1}{\gamma^2N}W=\text{const.}$$

p.28

Coupled envelope eqs. for a bunched beam

$$k_{x0}^2 a - \frac{3}{2}\frac{Nr_c}{\beta_0^2\gamma_0^3}\frac{1}{az_m}\left(1-\frac{g_0}{2}\frac{a^2}{\gamma_0^2 z_m^2}\right) - \frac{\varepsilon_{nx}^2}{\beta_0^2\gamma_0^2 a^3} = 0$$

$$k_{z0}^2 z_m - \frac{3}{2}\frac{Nr_c}{\beta_0^2\gamma_0^5}\frac{g_0}{z_m^2} - \frac{\varepsilon_{nz}^2}{\beta_0^2\gamma_0^6 z_m^3} = 0$$

$$g_0 \approx \frac{2\gamma_0 z_m}{3a}$$

space-charge dominated $\varepsilon=0$

$$k_{x0}^2 a - \frac{3}{2}\frac{Nr_c}{\beta_0^2\gamma_0^3}\frac{1}{az_m}\left(1-\frac{1}{3}\frac{a}{\gamma_0 z_m}\right) = 0$$

$$k_{z0}^2 z_m - \frac{Nr_c}{\beta_0^2\gamma_0^4}\frac{1}{az_m} = 0$$

p.38

$$\frac{z_m}{a} = \frac{2}{3\gamma_0}\left(\frac{k_{x0}^2}{k_{z0}^2}+\frac{1}{2}\right)$$

equipartitioning

$$\gamma_0\frac{\varepsilon_{nx}}{\varepsilon_{nz}}\frac{k_x}{k_z} = 1$$

$$\frac{k_{x0}}{k_{z0}} = \left(\frac{3}{2}\frac{\varepsilon_{nz}}{\varepsilon_{nx}} - \frac{1}{2}\right)^{1/2}$$

Acceleration process-1

$$\frac{k_{x0}}{k_{z0}} = \left(\frac{3}{2} \frac{\varepsilon_{nz}}{\varepsilon_{nx}} - \frac{1}{2} \right)^{1/2}$$

$$k_{z0} = \left(-\frac{q}{mc^2} \frac{2\pi}{\lambda} \frac{E_m \sin\phi_s}{\beta_0^3 \gamma_0^3} \right)^{1/2}$$

$$k_{z0} \propto \frac{1}{(\beta_0 \gamma_0)^{3/2}}$$

横の収束 σ_0=const

$$k_{x0} = \frac{\sigma_{x0}}{2\beta_0 \lambda}$$

$$k_{x0} \propto \frac{1}{\beta_0}$$

$$\frac{k_{x0}}{k_{z0}} \propto \beta_0^{1/2} \gamma_0^{3/2}$$

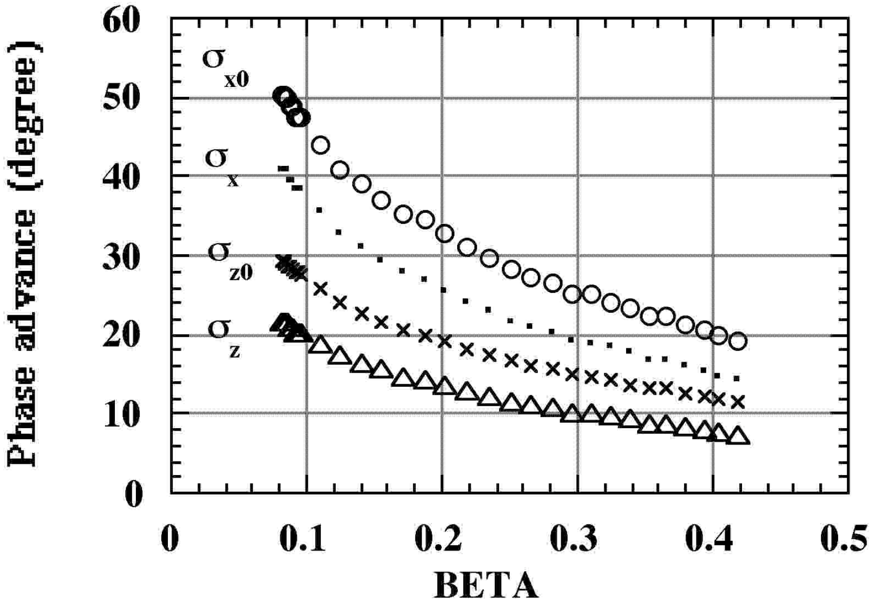
equipartitioning focusing

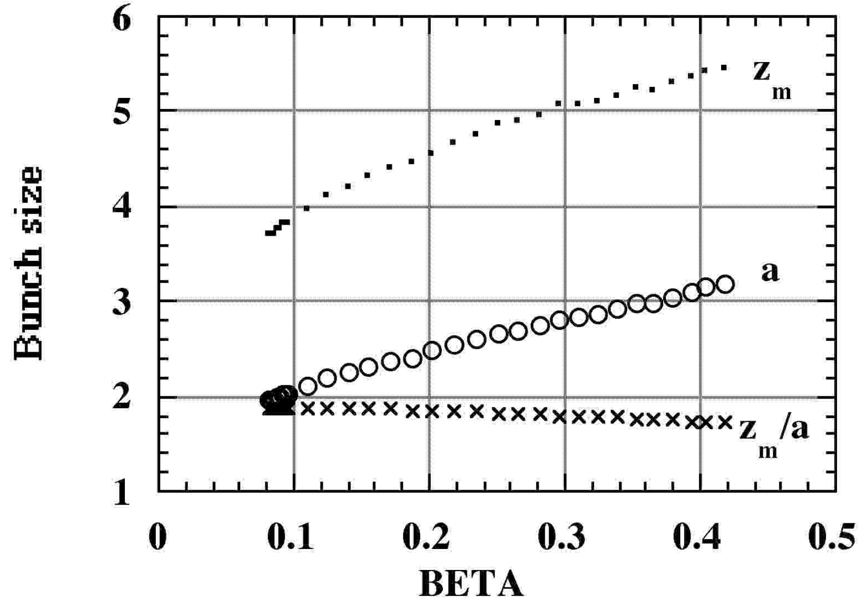
$$k_{x0} \propto \frac{1}{(\beta_0 \gamma_0)^{3/2}}$$

$$\sigma_{x0} \propto \frac{1}{\left(\beta_0 \gamma_0^3\right)^{1/2}}$$

p.38~

Acceleration process-2



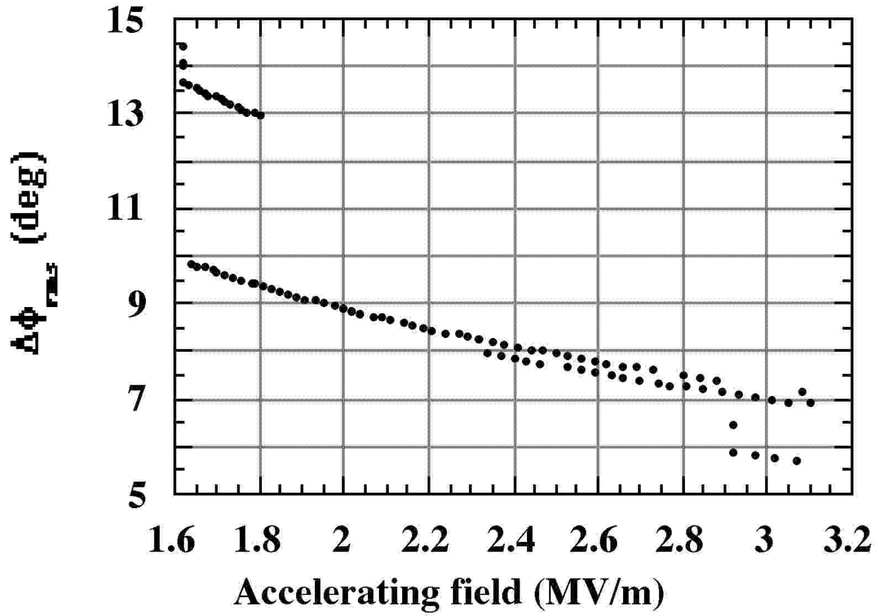


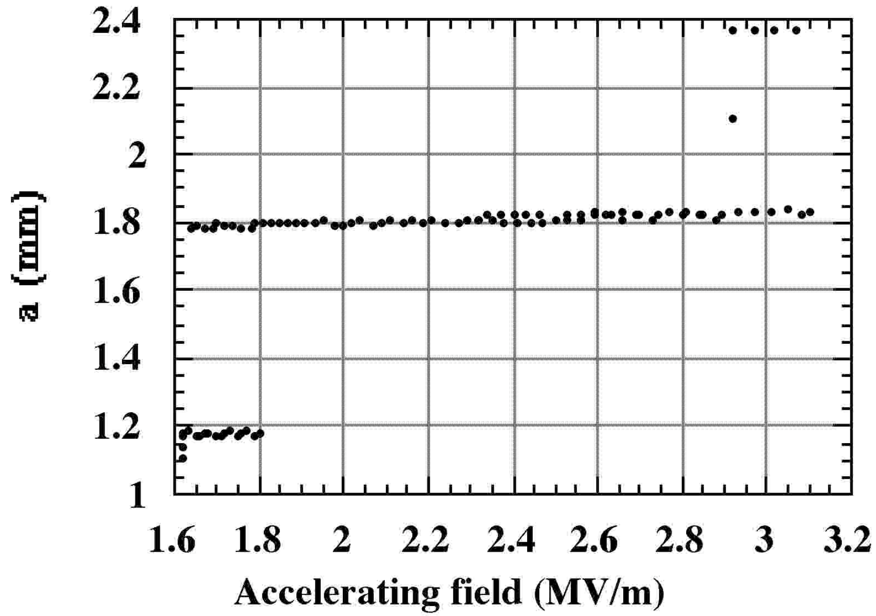
$$a = \left[\frac{\varepsilon_{nx}^2}{\varepsilon_{nz}^2}\frac{1}{k_{z0}^2}\frac{Nr_c}{\beta_0^2\gamma_0^2}\right]^{1/3} \propto \beta_0^{1/3}\gamma_0^{1/3} \qquad z_m = \left[\frac{\varepsilon_{nz}}{\varepsilon_{nx}}\frac{1}{k_{z0}^2}\frac{Nr_c}{\beta_0^2\gamma_0^5}\right]^{1/3} \propto \frac{\beta_0^{1/3}}{\gamma_0^{2/3}}$$

p.40

DTL injection-1

- matched
- equipartitioned





p.54

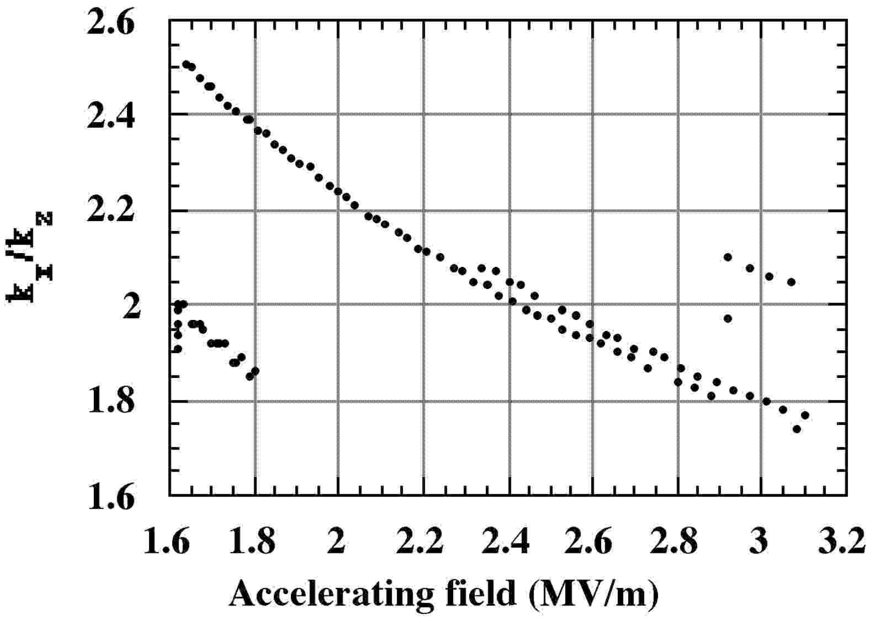
DTL injection-2

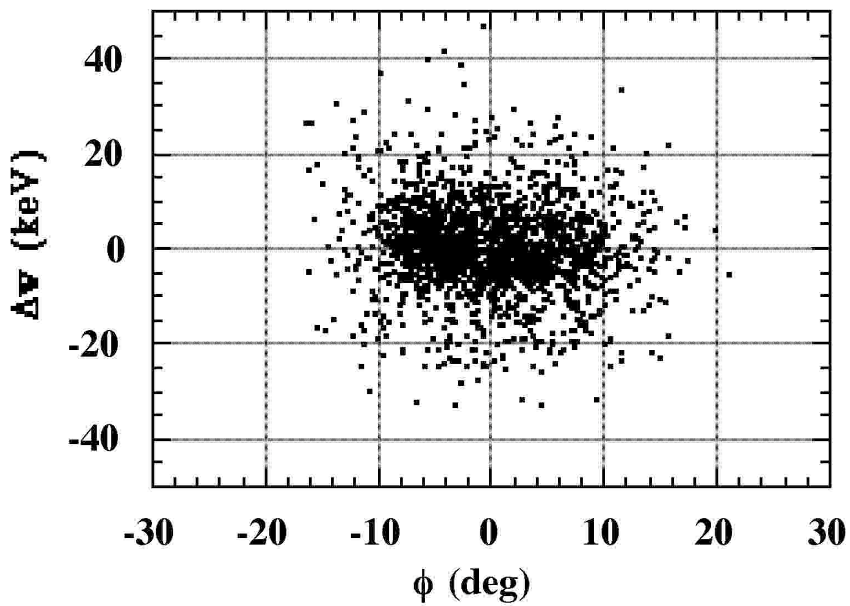
表6 - 4 matched and equipartitionedの条件を満たす入射パラメーターの計算例。

F	W	EZOPT	AM	DPHIRMS	P	DPBYP	ENXN	CI	BD	SIGZ0	SIGZ	SIGX	MUX	MUZ	KX/KZ	SCPX	SCI
f	w	E_m	a	$\Delta\phi_{rms}$	p	$\Delta p/p$	ε_{nx}	I	B'	σ_{z0}	σ_z	σ_x	k_x/k_{x0}	k_z/k_{z0}	k_x/k_z	μ_x	μ_z
324.	3.	3.03	1.82	7.3	.981	.0047	.15	.03	109.2	17.2	26.4	47.7	.82	.77	1.80	.32	.41
324.	3.	2.80	1.83	7.4	1.022	.0045	.15	.03	109.2	16.5	25.0	47.9	.83	.76	1.92	.32	.43
324.	3.	2.95	1.82	7.4	.982	.0047	.15	.03	109.2	16.9	26.1	47.8	.82	.77	1.83	.32	.41
324.	3.	2.73	1.82	7.5	1.025	.0045	.15	.03	109.2	16.3	24.6	47.9	.83	.76	1.95	.32	.43

p.54

DTL injection-3





p.55

JHP 200-MeV Proton Linac

· peak current	30 mA
· pulse length	400 μsec
· repetition rate	25 Hz
· rf max. duty factor	3%
· output energy	200 MeV
· negative hydrogen	

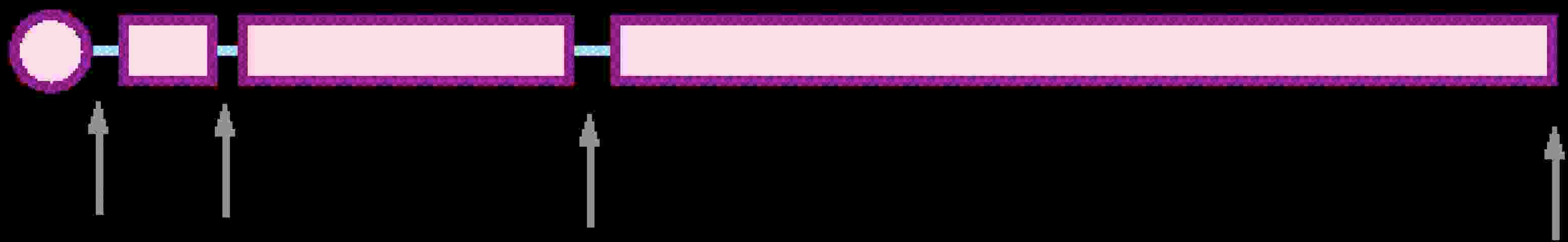
p.46

Fundamental parameters

- frequency
- in and out energies for each structure
- rf power source
- accelerating parameters
  - Eacc, stable phase
  - unit-tank length, total length
  - peak surface field, rf power
  - focusing criterion

JHP 200-MeV Proton Linac

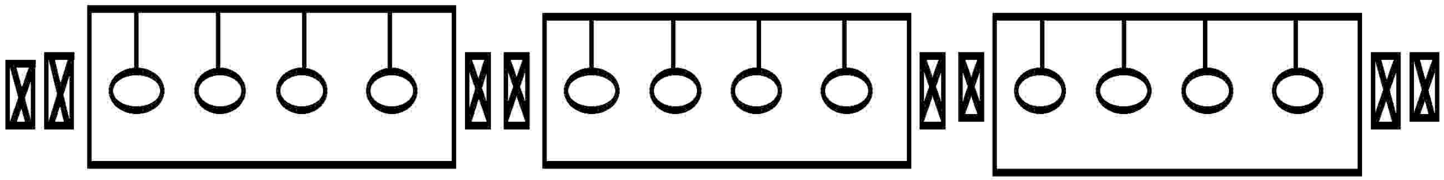
	RFQ	DTL	SDTL	
Frequency	324	324	324	MHz
Output energy	3	55	200	MeV
Length		32	122	m
Structure only		30	87.4	m
Number of tanks		4	41	
Number of cells		160	205	
Pc		4.3	10.2	MW



p.41

New structure concept SDTL

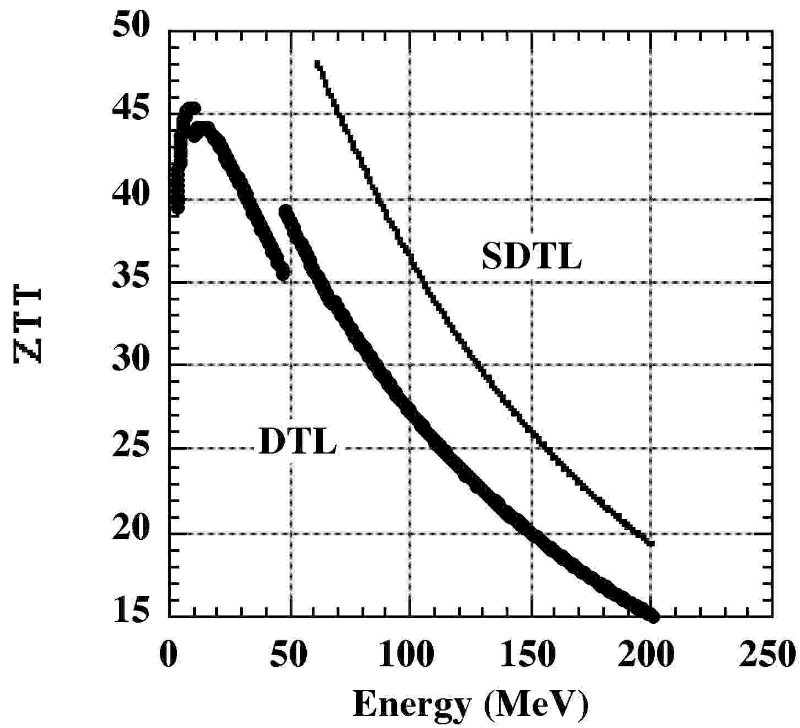
- DTL (difficulties, higher cost etc.)
  - Q-magnet inside of the drift tube
    - permanent Q-magnet
      - const. tune
    - take out Q-mags from drift tube
- SDTL (separated-type)



p.49

SDTL

- reduce construction cost
  - Q-magnet fabrication
  - DTL fabrication
  - alighnment
- increase shunt impedance

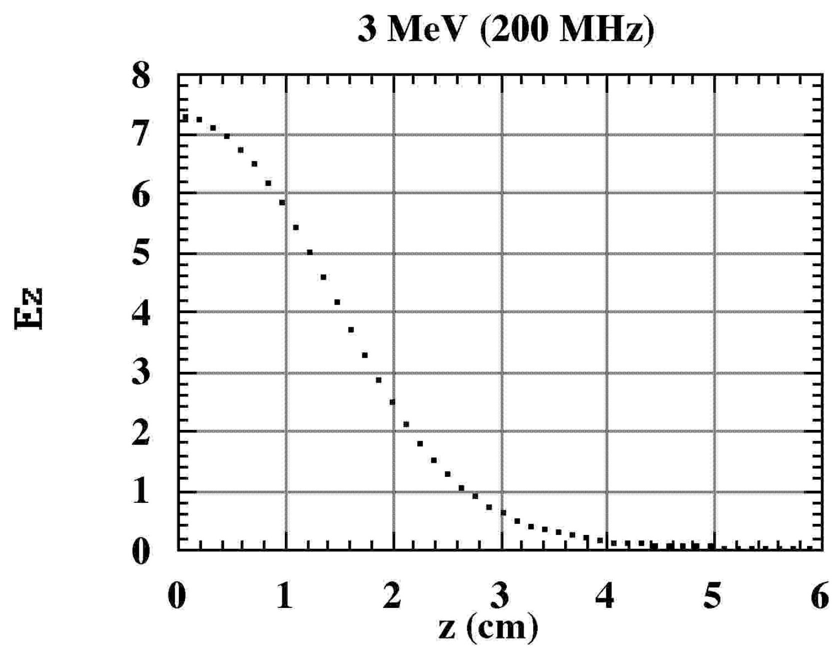


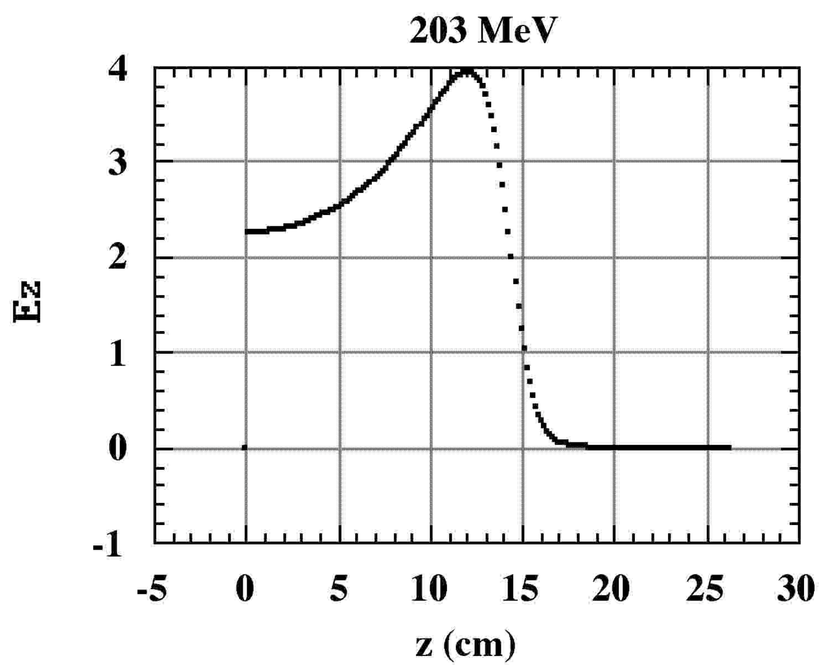
p.49

code LINSAC

- average accelerating field
- average Coulomb force

$$F_i = -\frac{\partial}{\partial \mathbf{r}} q\phi_s + \frac{q^2}{4\pi\varepsilon_0}\sum_j \frac{\mathbf{r}_{ij}}{r_{ij}^3}$$





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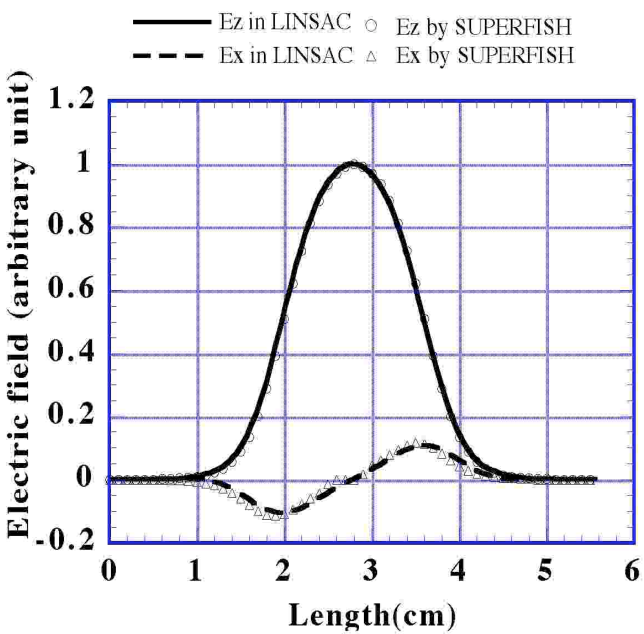
code LINSAC 2



$$\frac{d\mathbf{p}}{dt} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$E_z = \sum_{m=0}^{\infty} A_m I_0(k_m r) \cos\frac{2m\pi z}{L} \cos\omega t$$

$$E_r = \sum_{m=1}^{\infty} \frac{A_m 2m\pi}{k_m L} I_1(k_m r) \sin\frac{2m\pi z}{L} \cos\omega t$$

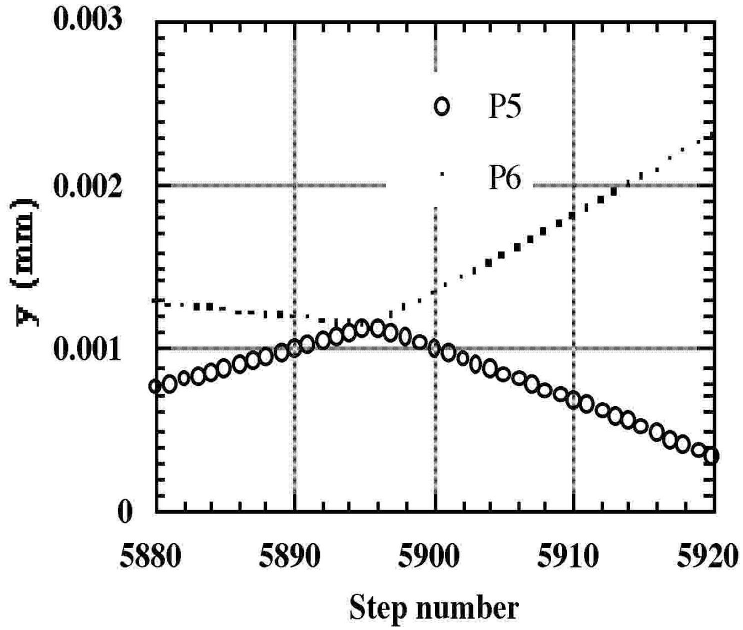
$$k_m^2 = \left(\frac{2\pi}{\lambda}\right)^2 \left(\left(\frac{m\lambda}{L}\right)^2 - 1\right)$$

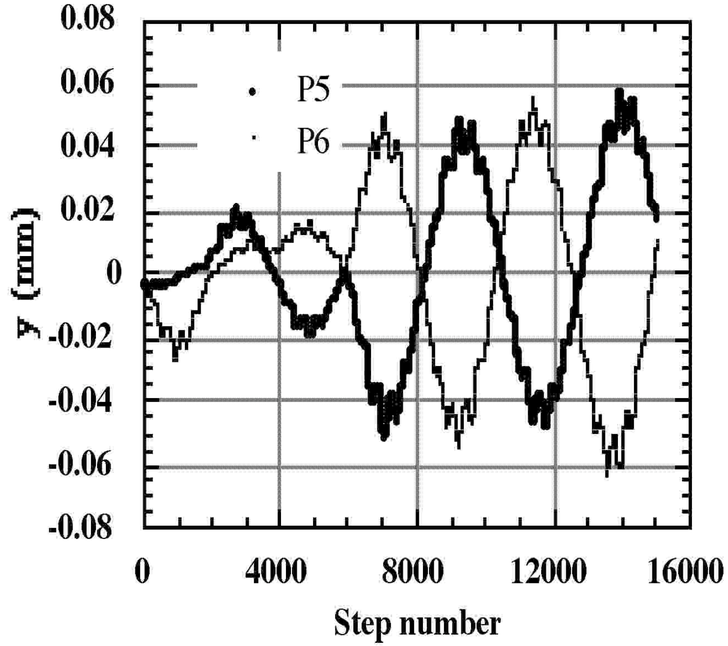
$$\mathbf{E}_i = \frac{q}{4\pi\varepsilon_0} \sum_{j\neq i}^{N} \frac{\mathbf{r}_{ij}}{r_{ij}^3}$$

360 cell, 181 step/cell, 48000 particle
Fujitsu Supercomputer VPP500
Vector-parallel 64 processor units
32 hours

p.59

Collision





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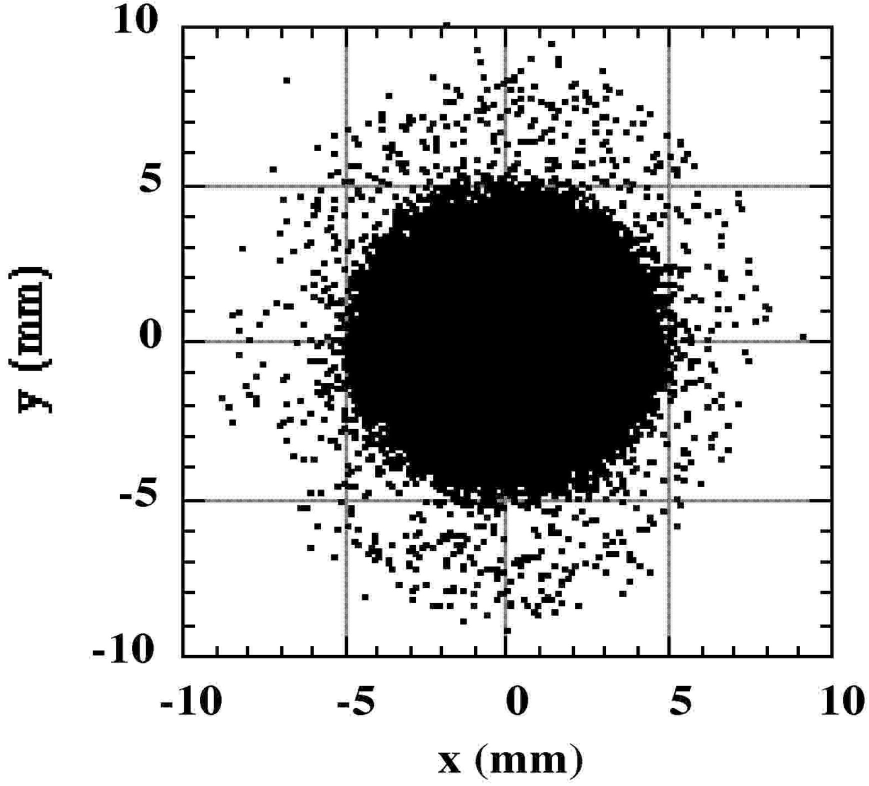
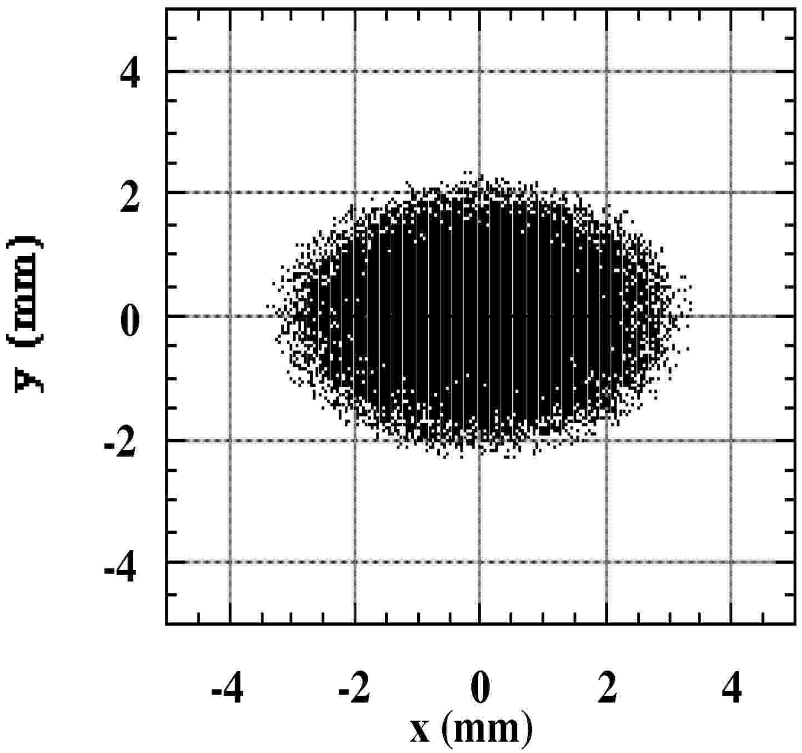
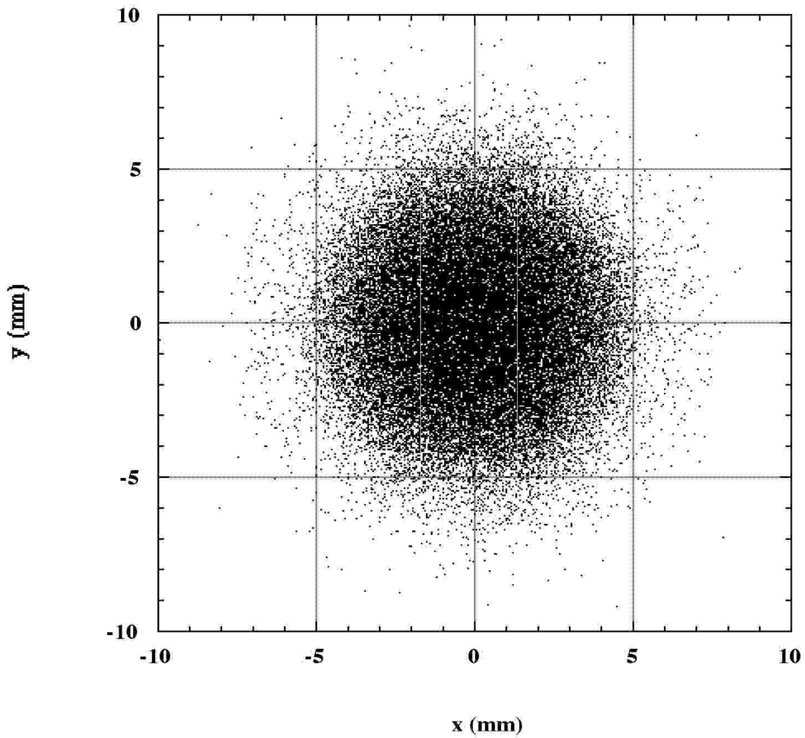
Rutherford scattering

Beam profile



Beam profile-2





r203

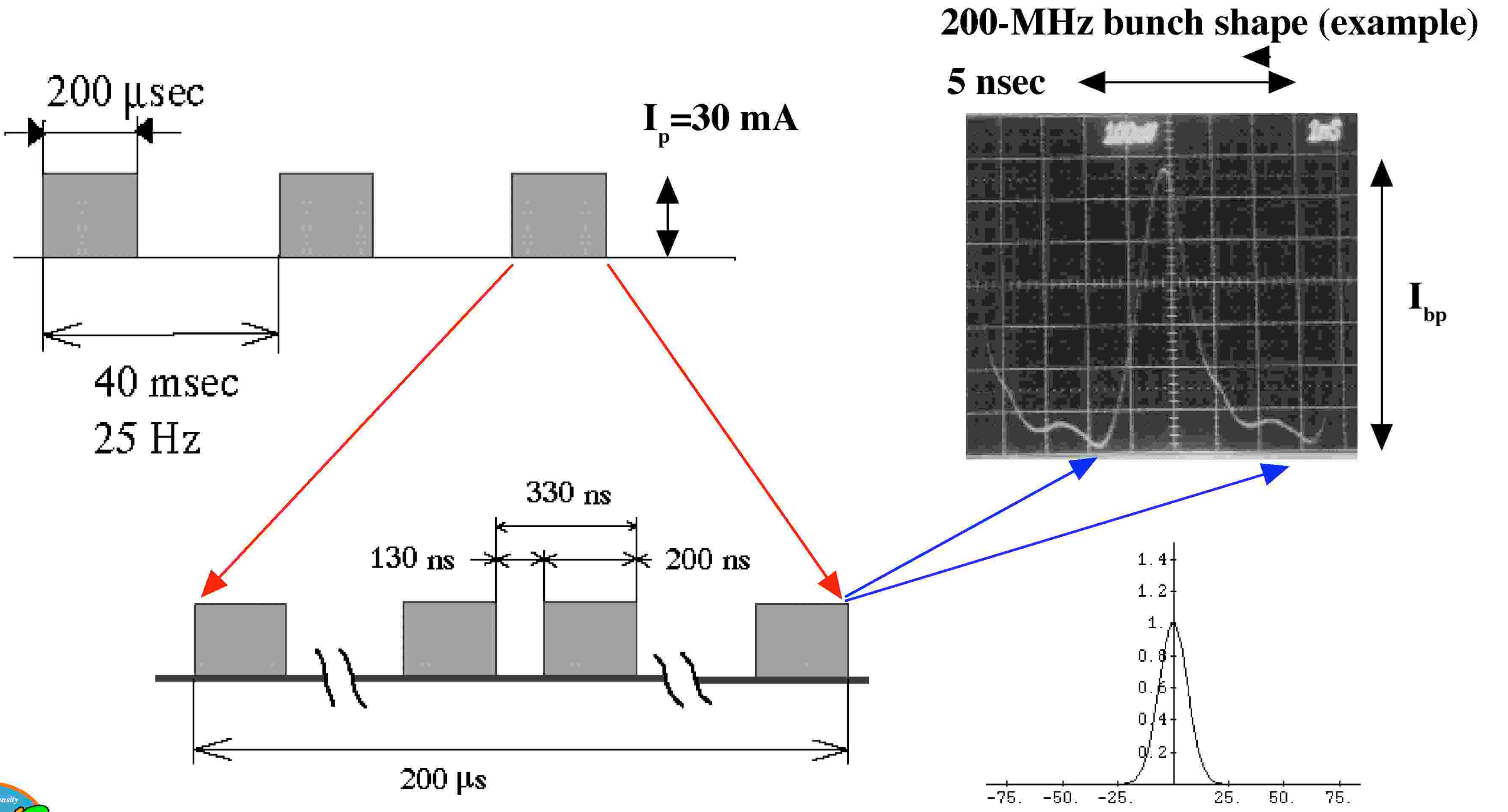
Equations without space-charge

$$\frac{d}{ds}\left(\beta_0\gamma_0\frac{dx}{ds}\right)=\frac{q}{mc^2}\left(\frac{-\pi}{\lambda}\frac{E_m\sin\phi}{\beta_0^2\gamma_0^2}-cB'\right)x$$

OHO '84 I- p.15 2.72 式

p.12

Beam structure



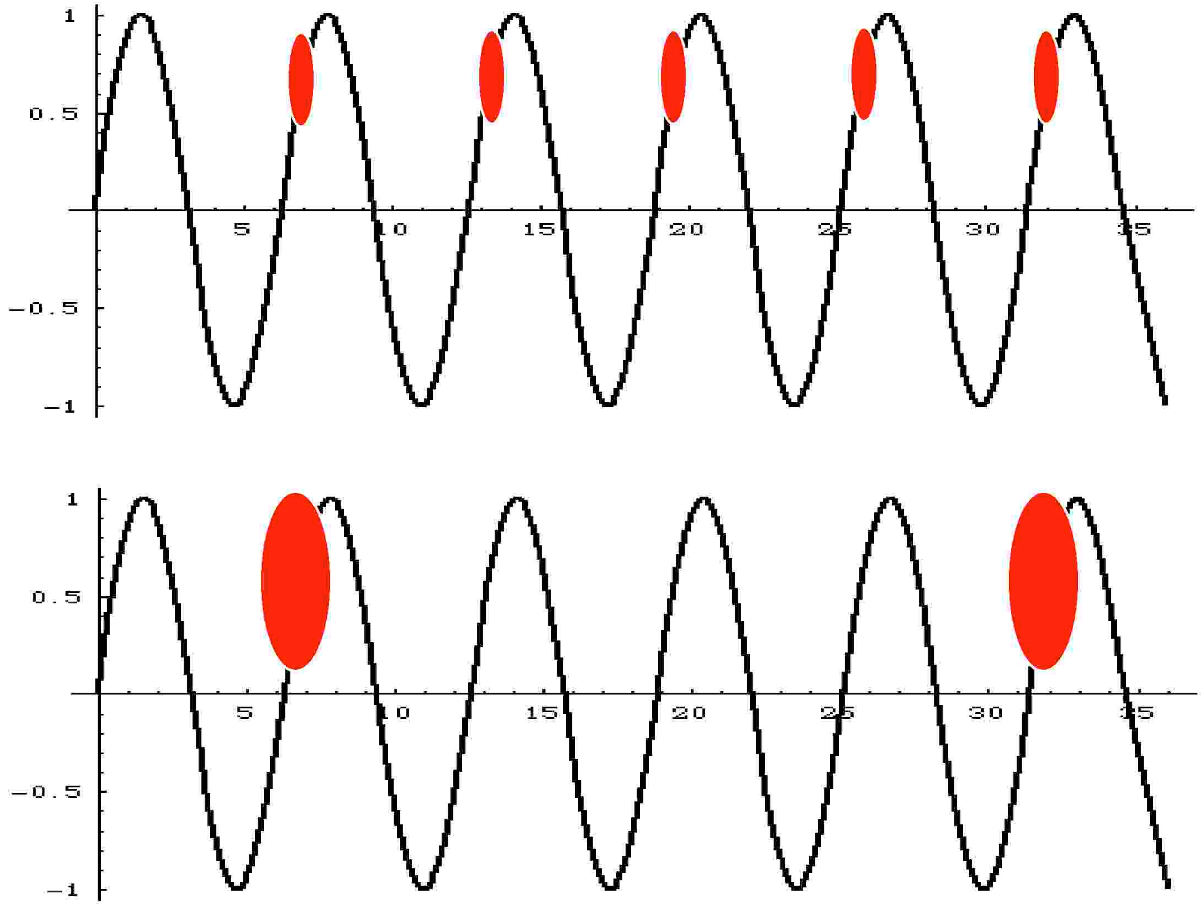
with rf fast chopper

degree

Beam loss

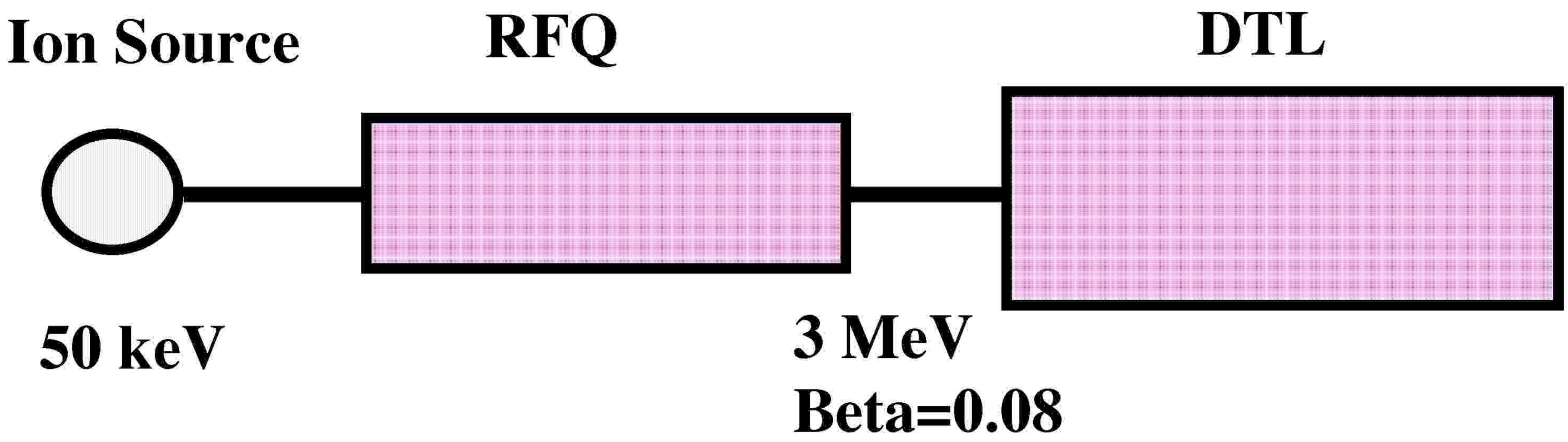
- Transient beam loss - beam loading
- Transition
- Space-charge effects
- Beam halo
- Errors
  - Fabrication
  - Alighnment
  - Tuning: rf amplitude, phase, Q-magnet

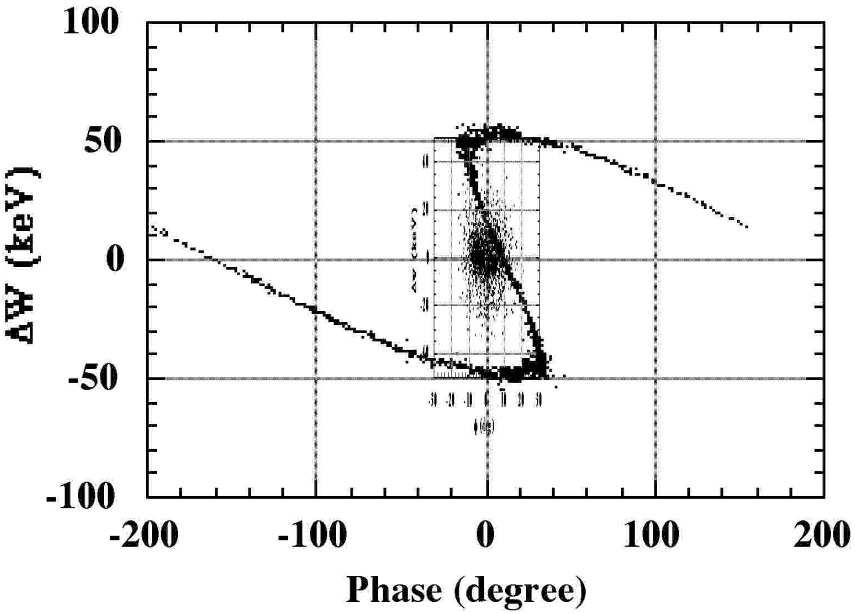
Number of particles / bunch



I=Nqf

RFQ impact


Ion Source
RFQ
DTL
50 keV
3 MeV
Beta=0.08


ΔW (keV)
100
50
0
-50
-100
-200
-100
0
100
200
Phase (degree)

Injection into DTL