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概要 (ABSTRACT)

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STABILITY OF THE STANDING-WAVE ACCELERATING STRUCTURE STUDIED WITH A COUPLED RESONATOR MODEL

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Abstract A coupled resonator model is used to study field stability of the 0, π, π/2, and 2π/3 mode standing-wave accelerating structures against effects due to beam loading and manufacturing imperfections. The beam loading is taken into account for the case that a phase shift due to beam loading is compensated by detuning a resonant frequency. Approximate analytical solutions are presented.

INTRODUCTION

As higher energy and higher beam current are required for proton linear accelerators, accelerating structures have to be more stable against high power dissipation and heavy beam loading. An alternating periodic structure (APS)[1,2] sometimes refered to as a biperiodic structure operated in a π/2 stading wave mode has relative immunity from the beam loading and detuning effects while sacrificing little in shunt impedance compared to π mode operation in a uniform periodic structure. A triperiodic structure[3] operated in a 2π/3 mode will have some advantages by eliminating one out of two coupling cells. In this report a coupled resonator model[4,5] extensively used to analyze behavior of the π/2 mode is extended to the 2π/3 mode and used to present analytic expressions, although approximate, for the effects due to the beam loading and manufacturing imperfections. The analytic expressions will be useful to choose a suitable structure for a specific accelerator and to make trade-off between possibly conflicting requirements.

COUPLED RESONATOR MODEL

Various physical systems are characterized by an equation

$$[\,j\,(\frac{\omega}{\omega_\mu} - \frac{\omega_\mu}{\omega}) + \frac{1}{Q_\mu}\,]\,x_\mu = -j\,\frac{\kappa}{2}\,(x_{\mu+1} + x_{\mu-1}) + \frac{R_\mu}{Q_\mu}\,i_\mu \quad , \tag{1}$$

which is refered to as a coupled resonator model[4,5] equation here. The parameter x_μ is proportional to a square root of a stored energy[6] of the μ-th oscillator or in the μ-th cell with a resonant frequency ω_μ. The second term of the l.h.s. represents a damping of the oscillator characterized by the quality factor Q_μ, the first term of the r.h.s. stands for couplings with the neighboring cells, and the last term is a driving force. From now on, suffices a, m and c, l instead of the suffix μ will be appended to parameters for the accelerating and coupling cells, respectively, and the cells will be numbered as the m-th accelerating cell or l-th coupling cell. Then, relations of m and l with μ are dependent upon a phase shift ß: $\mu = m$ for $ß = 0$ or π, $\mu = 2m - 1$ for $ß = \pi/2$, $\mu = [(m-1)/2] + m$ for $ß = 2\pi/3$, $\mu = 2l$ for $ß = \pi/2$, and $\mu = 3l$ for $ß = 2\pi/3$, where [y] denotes an integral part of y. To make $x_{a,m}$ positive we modify phases of $x_{a,m}$'s as $x_{a,m} = (-)^{m-1} x_\mu$ except for $ß = 0$, and for convenience phases of $x_{c,l}$'s are also modified as $x_{c,l} = (-)^{l-1} x_\mu$ and $x_{c,l} = -x_\mu$ for $ß = \pi/2$ and $2\pi/3$, respectively.

Further we introduce parameters defined by

$$\Delta_{a,m} = -\frac{1}{\kappa}\left(\frac{\omega}{\omega_{a,m}} - \frac{\omega_{a,m}}{\omega}\right) + \frac{1}{\kappa}\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right) - \cos ß \quad , \tag{2}$$

$$\Delta_{c,l} = -\frac{1}{\kappa}\left(\frac{\omega}{\omega_{c,l}} - \frac{\omega_{c,l}}{\omega}\right) - \cos ß \quad , \tag{3}$$

$$\delta_{a,m} = \frac{1}{\kappa Q_{a,m}} \quad , \tag{4}$$

$$\delta_{c,l} = \frac{1}{\kappa Q_{c,l}} \quad , \tag{5}$$

$$X_{a,m} = \left[1 + j\, Q_{a,m}\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)\right] x_{a,m} - R_{a,m}\, i_{a,m} \quad , \tag{6}$$

where ω_o is a resonant angular frequency of the accelerating mode. A phase relation between $i_{a,m}$ and i_μ is given by the same one between $x_{a,m}$ and x_μ. In the case of the $\pi/2$ mode the parameters $\Delta_{a,m}$ and $\Delta_{c,l}$ are identical to Δ_μ defined in Ref. 5 except for the second term of the r.h.s. of eq. (2). This term is introduced for convenience to take the beam loading effect into account together with the other terms than $x_{a,m}$ in eq.(6).

It is noted that the parameters $\Delta_{a,m}$ and $\Delta_{c,l}$ are approximately given by $\Delta_{a,m} \cong \Delta\omega_{a,m}/(\Delta\omega_p/2)$ and $\Delta\omega_{c,l} \cong \Delta\omega_{c,l}/(\Delta\omega_p/2)$, respectively, in the case of $|\omega - \omega_\mu| << \omega_\mu$, where $\Delta\omega_p = \kappa\omega_a$ is a passband width and $\Delta\omega_{a,m} = \omega_{a,m} - \omega_a$ and $\Delta\omega_{c,l} = \omega_{c,l} - \omega_a$ are deviations of the cell frequencies from the average value ω_a. Also $\delta_{a,m} = \delta\omega_{a,m}/(\Delta\omega_p/2)$ and $\delta_{c,l} = \delta\omega_{c,l}/(\Delta\omega_p/2)$, where $\delta\omega_{a,m} = \omega_a/(2Q_{a,m})$ and $\delta\omega_{c,l} = \omega_a/(2Q_{c,l})$ are half widths of the resonances or damping rates of the cells. Since the passband width is proportional to a group velocity, the parameters $\delta_{a,m}$ and $\delta_{c,l}$ are damping rates as measured by the group velocity.

In terms of the parameters introduced above, fields $x_{a,m}$ and $x_{c,l}$ can be expressed as follows[7]. For the 0 and π modes

$$x_{a,m} - x_{a,1} = \pm 2 \sum_{l=1}^{m-1} \sum_{n=1}^{l} (\Delta_{a,n} x_{a,n} + j\, \delta_{a,n} X_{a,n}) \quad . \tag{7}$$

Here, signs of $\pm$ correspond to the 0 and π modes, respectively. For the $\pi/2$ mode,

$$x_{c,l} = 2 \sum_{m=1}^{l} (\Delta_{a,m} x_{a,m} + j\, \delta_{a,m} X_{a,m}) \quad , \tag{8}$$

$$x_{a,m} - x_{a,1} = -4 \sum_{l=1}^{m-1} (\Delta_{c,l} + j\, \delta_{c,l}) \sum_{n=1}^{l} (\Delta_{a,n} x_{a,n} + j\, \delta_{a,n} X_{a,n}) \quad . \tag{9}$$

For the $2\pi/3$ mode,

$$x_{c,l} = 2 \sum_{m=1}^{2l} (\Delta_{a,m} x_{a,m} + j\, \delta_{a,m} X_{a,m}) \quad , \tag{10}$$

$$x_{a,m} - x_{a,1} = +2 \sum_{n=1}^{(m-1)/2} (\Delta_{a,2n} x_{a,2n} + j\, \delta_{a,2n} X_{a,2n})$$

$$-4 \sum_{l=1}^{(m-1)/2} (\Delta_{c,l} + j\, \delta_{c,l}) \sum_{n=1}^{2l} (\Delta_{a,n} x_{a,n} + j\, \delta_{a,n} X_{a,n}) \tag{11}$$

in the odd m case and

$$x_{a,m} - x_{a,1} = -2 \sum_{n=1}^{m/2} (\Delta_{a,2n-1} x_{a,2n-1} + j\, \delta_{a,2n-1} X_{a,2n-1})$$

$$-4 \sum_{l=1}^{m/2-1} (\Delta_{c,l} + j\, \delta_{c,l}) \sum_{n=1}^{2l} (\Delta_{a,n} x_{a,n} + j\, \delta_{a,n} X_{a,n}) \tag{12}$$

in the even m case. For the 0 and π modes, the parameters $\Delta_{a,1}$ and $\Delta_{a,N}$ of the end accelerating cells are modified to compensate effect of absence of cells beyond the end cells,where N is a number of the accelerating cells. Boundary conditions of $x_{c,o} = 0$ and $x_{c,N} = 0$ or $x_{c,N/2} = 0$ lead to

$$\sum_{m=1}^{N} (\Delta_{a,m} x_{a,m} + j\, \delta_{a,m} X_{a,m}) = 0 \quad . \tag{13}$$

Equations (8) and (9) for the $\pi/2$ mode are essentially the same as derived in Ref. 5. Equation (7) for the 0 and π modes can be considered as a special case of eq. (9) for the $\pi/2$ mode with $\delta_{c,l} = 0$ and $\Delta_{c,l} = \mp 1/2$. Thus, expressions for the 0 and π modes will be omitted from now on.

If the cavity is drived at the k-th accelerating cell by a driving current i_d that is defined real, then

$$i_{a,m} = i_d \delta_{m,k} + i_b e^{j\phi_m} \quad , \tag{14}$$

where i_b is a ω-component of the beam current and ϕ_m is a phase of the beam current at the m-th accelerating cell. Since the current $i_{a,m}$ is defined so as to drive the cavity, i_b is negative and related to the beam current i_o as $i_b = -2i_o$.

RESULTS OF FIRST-ORDER APPROXIMATION

In principle iterations for eqs. (7) – (12) may be made to obtain exact values of the parameters $x_{a,m}$ and $x_{c,l}$. However, our purpose is to obtain practically useful expressions for designing accelerating cavities and estimating their possible performances. For this purpose the first iteration, that is, the first-order approximation is accurate enough, where all of $x_{a,m}$'s in the r.h.s.'s of eqs. (7) – (12) are made identical to their average value x_a. Also, we assume $Q_{a,m} = Q_a (\delta_{a,m} = \delta_a)$, $Q_{c,l} = Q_c(\delta_{c,l} = \delta_c)$, $R_{a,m} = R_a$, and $\phi_m = \phi$.

In the absence of the beam loading a resonant frequency ω_o of a cavity will be tuned to a frequency ω of a driving current. In the presence of the heavy beam loading ω_o should be detuned from ω by an amount of

$$Q_a \left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right) x_a = R_a i_b \sin\phi \quad , \tag{15}$$

to keep $X_{a,m}$ real even in the presence of the beam. The detuning is advantageous for saving the RF power. Then, from eq. (6), (13) and (15) we obtain

$$\delta_{a,m} X_{a,m} = \delta_t x_a (1 - N \delta_{m,k}) \quad , \tag{16}$$

where δ_t is given by

$$\delta_t = \delta_a (1 + P_b/P_c) \quad , \tag{17}$$

P_b is beam power per cell given by $-(1/2)x_a i_b \cos\phi$, and P_c is power dissipation per cell given by $x^2{}_a/(2R_a)$. Effect of the beam loading is just decrease of the Q value, since the power loss in the accelerating cells arises from the beam loading P_b as well as the power dissipation P_c.

As seen from eqs. (7) – (12) the values of $x_{a,m} - x_{a,1}$ and $x_{c,l}$ are quite dependent upon the distributions $\Delta_{a,m}$ and $\Delta_{c,l}$. However, what we are interested in is the case that the maximum values of $x_{a,m} - x_a$ and $x_{c,l}$ are obtained for given tolerances. Suppose that the maximum values of $|\Delta_{a,m}|$ and $|\Delta_{c,l}|$ are given by $|\Delta_a|$ and $|\Delta_c|$, respectively. Then, the interesting case is that

$$\Delta_{c,l} = \Delta_c \qquad \text{for any } l \quad , \tag{18}$$

$$\Delta_{a,m} = \begin{cases} +\Delta_a & \text{for } 1 \leqq m \leqq N/2 \\ -\Delta_a & \text{for } N/2+1 \leqq m \leqq N \end{cases} \quad , \tag{19}$$

since effects of $\Delta_{a,m}$ and $\Delta_{c,l}$ are cumulative in eqs. (7) – (12). Variation of phase in eq. (19) is necessary from the condition (13). Substitutions of eqs. (16), (18) and (19) in eqs. (7) – (12) lead to values of $x_{a,m} - x_{a,1}$ and $x_{c,l}$, whose maximum values are given in Table I. Since the maximum values of $x_{a,m} - x_a$ and $x_{c,l}$ are dependent upon ratios of $\Delta_a\Delta_c$ to $\delta_a\delta_c$ or others, those of all terms in $x_{a,m} - x_a$ and $x_{c,l}$ are individually listed in the table, where the approximation of N >> 1 is used.

Table I
Maximum deviation $\delta x_a^{(i)}$ of the i-th term in $x_{a,m} - x_a$ and maximum value $x_c^{(i)}$ of the i-th term in $x_{c,l}$. Consider Δ_a and Δ_c as positive in this table. Terms to which a symbol j is attached represent phase diviations in radian. If tuners are installed at every M accelerating cells (a number of the tuners is N/M), $N\Delta_a$ should be replaced by $M\Delta_a$.

ß	$\pi/2$	$2\pi/3$
$x_c^{(1)}/x_a$	$N\Delta_a$	$N\Delta_a$
$x_c^{(2)}/x_a$	$jN\delta_t$	$jN\delta_t$
$\delta x_a^{(1)}/x_a$	$(1/2)(N\Delta_c)(N\Delta_a)$	$(1/4)(N\Delta_c)(N\Delta_a)$
$\delta x_a^{(2)}/x_a$	$(1/4)(N\delta_c)(N\delta_t)$	$(1/8)(N\delta_c)(N\delta_t)$
$\delta x_a^{(3)}/x_a$	$j(1/2)(N\delta_c)(N\Delta_a)$	$j(1/4)(N\delta_c)(N\Delta_a)$
$\delta x_a^{(4)}/x_a$	$j(1/4)(N\Delta_c)(N\delta_t)$	$j(1/8)(N\Delta_c)(N\delta_t)$
$\delta x_a^{(5)}/x_a$	—	$(1/2)N\Delta_a$
$\delta x_a^{(6)}/x_a$	—	$jN\delta_t$

If the fields $x_{c,l}$ in the coupling cells are excited, the total Q value Q_{tot} is decreased as

$$\frac{Q_a - Q_{tot}}{Q_{tot}} = \left(\frac{Q_a}{Q_c} - 1\right)\frac{\chi}{1+\chi} \quad , \tag{20}$$

where

$$\chi = (1/3)[(N\Delta_a)^2 + (N\delta_t)^2] \qquad \text{for } ß = \pi/2 \quad , \tag{21}$$

$$\chi = (1/6)[(N\Delta_a)^2 + (N\delta_t)^2] \qquad \text{for } ß = 2\pi/3 \quad . \tag{22}$$

Effect of tuners mentioned in Table I replaces $(N\Delta_a)^2$ by $MN\Delta_a^2$.

DISCUSSION

It is seen from Table I that the maximum deviations $\delta x_a^{(i)}$ and the maximum values $x_c^{(i)}$ are entirely expressed in terms of the parameters $N\Delta_a$, $N\Delta_c$, $N\delta_t$ and $N\delta_c$. The parameters $N\Delta_a$ and $N\Delta_c$ are the accelerating and coupling cell frequency deviations, respectively, divided by a half of the frequency difference between the $\pi/2$ mode and the neighbouring mode that is $\Delta\omega_p/(2N)$. The parameters $N\delta_t$ and $N\delta_c$ are ratios of half widths of resonances to $\Delta\omega_p/(2N)$. Thus, these four parameters can be measures of the deviations from the ideally periodic and perfectly conducting structure.

It is interesting to note that a pair of adjacent accelerating cells without coupling cell in between in the $2\pi/3$ mode corresponds to an accelerating cell in the $\pi/2$ mode. Coupling cells in the $2\pi/3$ mode stabilize stored energies of pairs of adjacent accelerating cells throughout a tank. Thus, the terms $\delta x_a^{(1)}$ to $\delta x_a^{(4)}$ of the $2\pi/3$ mode are the same as those of the $\pi/2$ mode except for the factor two as seen from Table I. It is also seen from eqs. (11) and (12) that the first terms are cancelled out by adding $x_{a,m}$ to $x_{a,m+1}$ with odd m, resulting in the same form as eq. (9) for the $\pi/2$ mode.

On the other hand the structure operated in the $2\pi/3$ mode has no apparatus to stabilize the fields between the two adjacent accelerating cells. This effect is expressed by the first terms in eqs. (11) and (12) or $\delta x_a^{(5)}$ and $\delta x_a^{(6)}$ in Table I. These terms cannot be eliminated by achieving the confluent condition $\Delta_c = 0$ as the $\pi/2$ mode. Here, it is interesting to compare $\delta x_a^{(1)} + \delta x_a^{(5)}$ of the $2\pi/3$ mode with $\delta x_a^{(1)}$ of the $\pi/2$ mode. The former becomes the same as the latter, if $N\Delta_c = 2$. Then, the advantage of the $\pi/2$ mode over the $2\pi/3$ mode would be lost. More physical implications of the results obtained above are detailed in Ref. 7.

The advantage of the triperiodic structure as revealed in Table I and eqs. (21) and (22) is that the number of coupling cells is a half of that of the APS. Thus, it will be worthwhile to consider the triperiodic structure in designing future proton linacs, keeping in mind characteristics of the $2\pi/3$ mode analyzed in this paper.

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